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Stress analysis of supporting rings for cylindrical shell loaded as beams

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STRESS ANALYSIS OF SUPPORTING RINGS

FOR CYLINDRICAL SHELLS LOADED AS BEAMS

by [illegible]

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This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

May 25, 1964
(date)

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Head of the Department

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ABSTRACT

Formulas were developed for calculating the bending moments, bending stresses, and radial deformations in rings supporting thin-walled, circular tubes in bending. Specific cases considered included: (1) A concentrated radial load applied to the ring in the plane of tube bending, and (2) Tangential and radial loads applied to the ring at two points located 30 degrees on either side of the plane of bending. Experimental verification of the formulas was sought; electric resistance strain gages were used to measure strains in a Plexiglas-plastic model.

Good agreement between theoretical and experimental stresses was obtained for case (1). For case (2), good agreement was obtained for all points around the ring except in the 60 degree angular region between the loads. In this case , the experimentally determined stresses were much lower than the theoretical values.

Theoretical and experimental deformations compared satisfactorily for both cases except at large deformations where non-linear behavior of the plastic model became significant.

SYMBOLS

A	AREA IN SQUARE INCHES
D	DIAMETRICAL CHANGE IN INCHES
E	MODULUS OF ELASTICITY IN PSI.
F	LONGITUDINAL FORCE IN LBS.
G	MODULUS OF RIGIDITY IN PSI.
I	MOMENT OF INERTIA IN $INCH^4$
J	TRANSVERSE SHEAR FORCE IN LBS.
M	MOMENT IN LBS. INCH
N	NORMAL FORCE IN LBS. PER INCH
P	LOAD IN LBS.
Q	RADIAL SHEAR FORCE IN LBS.
R	RADIUS IN INCH AND/OR CONSTANT
S	SHEAR FORCE IN LBS. OR LBS./UNIT LENGTH
T	TANGENTIAL FORCE IN LBS.
U	RADIAL CHANGE OF RING IN INCHES
V	DIRECT FORCE IN LBS.
W	WEIGHT OR LOAD IN LBS.
b	WIDTH OF THE RING
e	ECCENTRICITY OF NEUTRAL AXIS IN CURVED BEAM
f	FLEXURE STRESS IN PSI.
h	HEIGHT OF THE RING
i	SUBSCRIPT
t	THICKNESS OF THE RING
α	ANGLE OF REFERENCE AND/OR SUBSCRIPT

SYMBOLS (continued)

δ ANGLE BETWEEN NORMAL COMPONENT OF THE REACTION AND THE RESULTANT REACTION AT SUPPORT

θ ANGLE IN DEGREE

ϕ ANGLE OF REFERENCE FROM ANGLE α AND/OR SUBSCRIPT

ν POISSON'S RATIO

$$\mu = \frac{-\beta' + k\beta'' + \alpha' r^2}{\beta' + k\beta'' + \alpha' r^2} = 1 \quad \text{(in our case, see Ref. 2)}$$

where

$$\alpha' = \frac{1}{EI}$$

$$\beta' = \frac{1}{EA}$$

$$\beta'' = \frac{1}{GA}$$

σ_o STRESS AT OUTER SURFACE OF RING IN PSI.

γ ANGLE OF INCLINATION OF SUPPORTS FROM VERTICAL AXIS OF THE RING.

I. INTRODUCTION

The aim of this investigation is to develop formulas giving the stresses and the deflections in rings used as support for cylindrical shells in bending. The configuration is shown in Figure 1.

This problem arises in various industrial applications such as cement kilns, pipe lines and high chimneys.

Section II of this report gives the development of the mathematical equations for the moments, stresses and deflections at any point on the ring.

Section III gives the experimental investigation including the design of the test model, the experimental procedure and the results obtained with different kinds of loadings.

In Section IV of this report the theoretical and experimental results for stresses and deflections are compared and discussed.

Detailed theoretical analysis and calculations will be found in the Appendixes, Section VII.

II. THEORETICAL ANALYSIS

This section includes a description of the analytical model and presents the theoretical equations developed for calculating the moments, stresses and deflections in the ring. Detailed development of the equations is given in the appendixes.

A. Loads Acting on the Ring

The forces acting on the ring are shown in Figure 2. They include the normal and tangential components of the reactions at the supports and the pressure of the shell acting on the inside circumference of the ring. The latter pressure arises from the shear forces, induced by the bending moment on the shell, acting on transverse sections of the shell located on either side of the ring.

The shearing forces may be calculated from the general theory of flexure provided the transverse section of the shell remains circular and the circumferential compression, due to a combination of large shear and narrow supporting rings, is not excessive (Ref. 1). Using the flexure theory, the transverse shear disbribution was found to be (see Appendix A):

$$S = \frac{J \sin\theta}{\pi r} \qquad \text{—————(1)}$$

The circumferential pressure of the shell on the ring may be calculated by assuming that this pressure holds the transverse shear forces in equilibrium. This calculation is given in Appendix A. The resulting pressure N_{α} shown in Figure 3 was found to be:

$$N_{\alpha} = \frac{P}{\pi r}\left[1-\cos\theta\right] \qquad \text{(2)}$$

If the total load on the ring is represented by P and the location of each support is given by an angle γ measured from the vertical axis as shown in Figure 2, then, assuming symmetry, the reaction at each support becomes $\frac{P}{2\cos(\delta - \gamma)}$.

Each reaction may be resolved into two components:

1. Radial component $\frac{P\cos\delta}{2\cos(\delta - \gamma)}$.

2. Tangential component $\frac{P\sin\delta}{2\cos(\delta - \gamma)}$.

B. Circular Ring Loaded in its Own Plane and Supported in a Statically Determinate Manner

The method of Biezeno and Grammel (Ref. 2) was applied for determining the moments in any section of the ring.

The ring, Figure 4, is sectioned at an arbitrary cross-section $\phi = 0$ when $\alpha = \alpha$. Our reference point is at top of the ring or $\alpha = 0$. The external loading consists of radial force $\frac{P\cos\delta}{2\cos(\delta - \gamma)}$ at $(\pi - \gamma)$ and $(\pi + \gamma)$, respectively, tangential force $\frac{P\sin\delta}{2\cos(\delta - \gamma)}$ at $(\pi - \gamma)$ and $(\pi + \gamma)$, respectively, and normal pressure $N_\alpha\,ds$.

We now consider the ring under the given external forces and develop the so-called "reduced" loading system of Biezeno and Grammel.

At section $\phi = 0$ apply the inner forces and moments V_o, Q_o, M_o, V_α, Q_α and M_α. The "reduced" loading system is obtained as follows:

1. Each force and moment is multiplied by the coefficient $\frac{\phi_i}{2\pi}$ corresponding to its position. The quantities V_o, Q_o and M_o acting on the right face of the section

at $\phi = 0$ thus vanish, whereas V_{oc}, Q_{oc} and M_{oc} on its left face remain undiminished. In the present case we will consider only the bending moment M_{oc}.

2. The "reduced" external radial force $\left[\frac{\phi_1}{2\pi} \frac{P\cos\delta}{2\cos(\delta-\gamma)}\right]$ is accompanied by a moment $-\frac{r}{2\pi} \cdot \frac{P\cos\delta}{2\cos(\delta-\gamma)}$ and the "reduced" external radial force $\int_0^{2\pi} \frac{\phi}{2\pi} N_{oc}\,ds$ is accompanied by a moment $-\int_0^{2\pi} \frac{r}{2\pi} N_{oc}\,ds$.

3. Each "reduced" external tangential force $\frac{\phi_1}{2\pi} \frac{P\sin\delta}{2\cos(\delta-\gamma)}$ is accompanied by a radial force $-\frac{\mu}{2\pi} \frac{P\sin\delta}{2\cos(\delta-\gamma)}$ at the same position ϕ_1.

4. There are no external moments.

The "reduced" loading system shown in Figure 5 is in the equilibrium, consequently the unknown moment M_{oc} acting on the left face of the section $\phi = 0$ is obtained by summing moments about the point of $\phi = 0$.

Moment equations for various support locations are given in the next section.

C. Equations for Moment and Deflection for Various Support Locations

In this section moment and deflection equations for various support location are presented.

1. Reaction having normal and tangential components.

a. Moment Equation

The forces acting are shown in Figure 2 and the "reduced" system for it is shown in Figure 5.

The resulting moment at any section on the ring was found to be (see Appendix B):

$$M_{\alpha} = -\frac{Pr}{2\pi}\left[\frac{\cos\delta}{\cos(\delta-\gamma)} - \frac{1}{2}\cos\alpha - \pi\sin\alpha\right.$$

$$+ \frac{\cos\delta}{\cos(\delta-\gamma)}\left(\pi\cos\gamma\sin\alpha - \gamma\sin\gamma\cos\alpha - \alpha\cos\gamma\sin\alpha\right)$$

$$- \mu\frac{\sin\delta\sin\gamma\cos\alpha}{\cos(\delta-\gamma)}$$

$$+ \frac{\sin\delta}{\cos(\delta-\gamma)}\left(\gamma + \pi\sin\gamma\sin\alpha + \gamma\cos\gamma\cos\alpha\right.$$

$$\left.\left.-\alpha\sin\gamma\sin\alpha\right)\right] \qquad (3)$$

b. Deflection Equation

Due to the bending moment M_α, the shape of the supporting ring will change. The relationship between the radial deformation of the ring and the bending moment is given by the following equation (Ref. 5):

$$\frac{d^2U}{ds^2} + \frac{U}{r^2} = \frac{M_\alpha}{EI} \qquad (4)$$

The sign of the bending moment is taken to be positive when it produces a decrease in initial curvature of the ring.

From equations 3 and 4

$$\frac{d^2U}{d\alpha^2} + U = -\frac{\alpha' P r^3}{2\pi}\Bigg[\frac{\cos\delta}{\cos(\delta-\gamma)} - \frac{1}{2}\cos\alpha$$

$$-\pi\sin\alpha + \frac{\cos\delta}{\cos(\delta-\gamma)}\big(\pi\cos\gamma\sin\alpha - \gamma\sin\gamma\cos\alpha$$

$$-\alpha\cos\gamma\sin\alpha\big) - \frac{\mu\sin\delta\sin\gamma\cos\alpha}{\cos(\delta-\gamma)}$$

$$+\frac{\sin\delta}{\cos(\delta-\gamma)}\big(\gamma + \pi\sin\gamma\sin\alpha + \gamma\cos\gamma\cos\alpha$$

$$-\alpha\sin\gamma\sin\alpha\big)\Bigg] \qquad (5)$$

On solving equation (5) for U (see Appendix C) we get:

$$U = K\left\{R\cos\alpha + \left[\frac{\pi}{2} - \frac{\pi\cos\delta\cos\gamma}{2\cos(\delta-\gamma)} - \frac{\pi\sin\gamma\sin\delta}{2\cos(\delta-\gamma)}\right]\sin\alpha\right.$$

$$- \left[\frac{\cos\delta}{\cos(\delta-\gamma)} - \frac{\alpha}{4}\sin\alpha + \frac{\pi\alpha}{2}\cos\alpha + \frac{\cos\delta}{\cos(\delta-\gamma)}\right.$$

$$\left(-\frac{\pi\alpha}{2}\cos\gamma\cos\alpha - \frac{\gamma\alpha}{2}\sin\gamma\sin\alpha + \frac{1}{4}\alpha^2\cos\gamma\cos\alpha\right.$$

$$\left.-\frac{1}{4}\alpha\cos\gamma\sin\alpha\right) - \frac{\mu\alpha}{2}\,\frac{\sin\delta\sin\gamma\sin\alpha}{\cos(\delta-\gamma)}$$

$$+\frac{\sin\delta}{\cos(\delta-\gamma)}\left(\gamma - \frac{\pi\alpha}{2}\sin\gamma\cos\alpha + \frac{\gamma\alpha}{2}\cos\gamma\cos\alpha\right.$$

$$\left.\left.\left.+\frac{\alpha^2}{4}\sin\gamma\cos\alpha - \frac{1}{4}\alpha\sin\gamma\sin\alpha\right)\right]\right\} \qquad (6)$$

Where $K = \dfrac{Pr^3}{2\pi EI}$

2. Normal Reactions

a. Moment Equation

We get the moment equation for the present case by substituting $\delta = 0$ in equation (3).

$$M = -\frac{Pr}{2\pi}\left[\frac{1}{\cos\gamma} - \cos\alpha\left(\frac{1}{2} + \gamma\tan\gamma\right) - \alpha\sin\alpha\right] \qquad (7)$$

b. Deflection Equation

On substituting $\delta = 0$ in equation (6) we get:

$$U = K\left[R\cos\alpha - \frac{1}{\cos\gamma} - \frac{\alpha^2}{4}\cos\alpha + \frac{\alpha}{2}\sin\alpha\,(1+\gamma\tan\gamma)\right] \qquad (8)$$

3. Single, Centrally Located Reaction

a. Moment Equation

In this case the load is vertical and passes through the center of the ring as shown in Figure 7. The moment equation for the present case can be obtained by substituting $\gamma = 0$ and $\delta = 0$ in equation (3). The result is:

$$M = -\frac{Pr}{2\pi}\left[1-\frac{1}{2}\cos\alpha - \alpha\sin\alpha\right] \qquad (9)$$

b. Deflection Equation

Substituting $\gamma = 0$ and $\delta = 0$ in equation (6) we get:

$$U = K\left[R\cos\alpha - 1 - \frac{\alpha^2}{4}\cos\alpha + \frac{\alpha}{2}\sin\alpha\right] \qquad (10)$$

D. Theoretical Formula for Bending Stresses in the Ring

The curved beam formula was used to calculate the bending stresses in the ring. This formula may be written as follows:

$$\sigma_o = -\frac{M_\alpha \left(\frac{h}{2} + e\right)}{A\, e\, r_o} \qquad (11)$$

(See also Appendix-D).

III. EXPERIMENTAL INVESTIGATION

This section includes the experimental approach for determining the stresses and deflections in the supporting ring. It describes the experimental model and the experimental procedure. Also, the experimental approach for finding the modulus of elasticity of the model material is described.

A. Experimental Model

The model was prepared from Plexiglas plastic. The plastic model was used because of availability, ease of manufacture and to provide measurable deformation.

To obtain a snug fit between the ring and the shell, the ring was heated by immersing it in hot water and then sliding it over the shell.

The dimensions of the experimental model follow (see Figure 1):

1. Shell
   - a. Total length of shell...........51.25"
   - b. Inside diameter.................9.75"
   - c. Outside diameter................10.00"
   - d. Thickness of shell..............0.125"

2. Ring

a. Width of ring..................2.00"

b. Inside diameter of ring........10.00"

c. Outside diameter of ring.......11.274"

d. Thickness of ring.............0.637"

3. Cylindrical Supports

a. Width.........................2.375"

b. Outside diameter..............2.75"

The supports for the end of the shell were made from maple wood (Figure 14). Steel plates were riveted on the bearing surfaces. Each support was clamped to the shell by one wooden ring and two aluminum bands to prevent splitting of the shell at the supports.

A piece of steel tubing, $1\frac{1}{2}$" in diameter, and a 90° angle channel were used for rotational and hinged end conditions. The distance between the two supports was 56".

B. Experimental Method for Determining the Modulus of Elasticity

A 2" wide ring segment was cut from the shell and its maximum concentrated load carrying capacity was calculated. The ring was loaded as shown in Figure 8. The loading device consisted of a steel wire hook and a piece of steel bar.

The load was dead weight. Using an outside micrometer, the horizontal diameter of the ring was measured at about 60 second intervals over a period of 12 minutes. For each increment of load, the diameter and time were recorded. Room temperature was also recorded.

The graph of time-deflection (Figure 15) shows that initial creep in the plastic is not significant after a "setting" period of about 10 minutes. The modulus of elasticity was determined from load-deflection values corresponding to a time period of 12 minutes (see Figure 16).

For the kind of loading present in this test the following formula (Ref. 4) gives the relationship among the load W, diametral deflection D, radius R and Modulus of Elasticity E:

$$D = 0.137 \frac{WR^3}{EI}$$

From Figure 16 we can find the load and deflection and by knowing the moment of inertia I for the cross-section of the ring and mean radius R we can find the unknown Modulus of Elasticity E. The experimentally determined value of E was 494000 psi.

C. Experimental Procedure.

Eight (8) strain gages were located at $\alpha = 0, 45, 90, 135, 180, -135, -90,$ and -45 degrees as shown in Figure 1. For the test having the single, centrally located load configuration, the strain gages were located at $\alpha = 19, 64, 109, 154, -26, -71, -116$ and -161 degrees as shown in Figure 9.

SR-4 strain gages were used having the following specifications:

Manufacturer:	Baldwin-Lima-Hamilton Corp. Electronic & Instrumentation Div. Waltham, Mass.
Type:	AR-5
Resistance:	120.0 ±0.2 OHMS
Gage Factor:	2.03 ±1%
Lot No.:	B-31

Dupont "Duco" cement was used for mounting the gages. To roughen the surface of the specimen, 180 grit emery paper was used, then the surfaces were cleaned with trichlorethylene.

One dummy or compensating strain gage was mounted on a piece of plastic. All strain gages were allowed to dry at room temperature for at least 24 hours.

A Baldwin-Southwark Universal Testing machine was used to apply load to the model. Figure 14 shows the model located in the machine.

To measure strain, a Baldwin-Lima-Hamilton Type 20 Strain Gage Indicator was used. A time period of 12 minutes was allowed between applying a load increment and reading the strain indicator. This permitted the initial creep phenomena to cease. Experimental stresses were calculated from the strain gage readings using the following formula:

Stress = (Final Reading in microinch/inch -Initial Reading in microinch/inch) XE

In our case E = 0.494×10^6 psi, therefore,

Stress = (Final Reading in inch/inch -Initial Reading in inch/inch)$10^{-6} \times 0.494 \times 10^6$

= 0.494 x (Final Reading in inch/inch -Initial Reading in inch/inch). PSI.

Diametral deformations along the horizontal diameter of the ring were determined using an outside micrometer, hence:

Diametral Change = (Final Micrometer reading in inch -Initial Micrometer reading in inch).

IV. DISCUSSION OF RESULTS

A. Single, Centrally Located Reaction.

Stresses

From Table 1 and Figures 17, 18, 19 and 20, it is apparent that the theoretically and experimentally determined stresses are in good agreement. From this we may conclude that the assumed mathematical model is useful for calculating moments and stresses for the single, centrally located load configuration.

Deformation Along Horizontal Diameter of the Ring

The comparison of theoretical and experimental results, Table II and Figure 21, shows increasing discrepancies between the linear-theoretical and non-linear-experimental deformation behaviors. Since the experimentally and theoretically determined bending stresses agree at the higher loads, we may conclude that the theoretical equation for bending moments in the ring is correct. The disagreement in the deformation results arises from the linearization process leading to the differential equation used in this analysis (Ref. 5). Because the deformation in the plastic becomes large, non-linear behavior is to be expected. Using the conditions of structural similarity it can be shown that the deformation of many practical, metallic prototypes are small, hence may be accurately predicted using the linear theory.

B. Supports Located at 30° With the Vertical.

Stresses

From the comparison of the theoretical and experimental results for stresses, Table III and Figures 22, 23, 24, 25, 26 and 27, it is apparent that agreement was obtained everywhere but in the region between the supports ($\alpha = 180°$).

In an effort to explain the discrepancy in this region, the influence of tangential components of the support reaction on the stress state was investigated. These tangential components may arise from the presence of friction between the ring and the fixed cylindrical supports. Because these tangential components are unknown, the resultant support reaction was assumed to act at some angle δ with the normal to the ring. It was hoped that proper selection of the value of angle δ would lead to theoretical and experimental stress agreement at all points around the ring. Such agreement did not result. We must conclude that the presence of tangential force at the supports cannot explain the lack of agreement in the region between the supports and that the mathematical model assumed for this analysis is unreliable for predicting stresses in this region.

Deformation Along Horizontal Diameter of the Ring

The comparison of experimentally and theoretically determined deformation is shown in Table IV and Figure 28. The discrepancies arise due to the non-linear behavior of the plastic model at the higher loads. Remarks made previously for the single, centrally located reaction also apply to the present case.

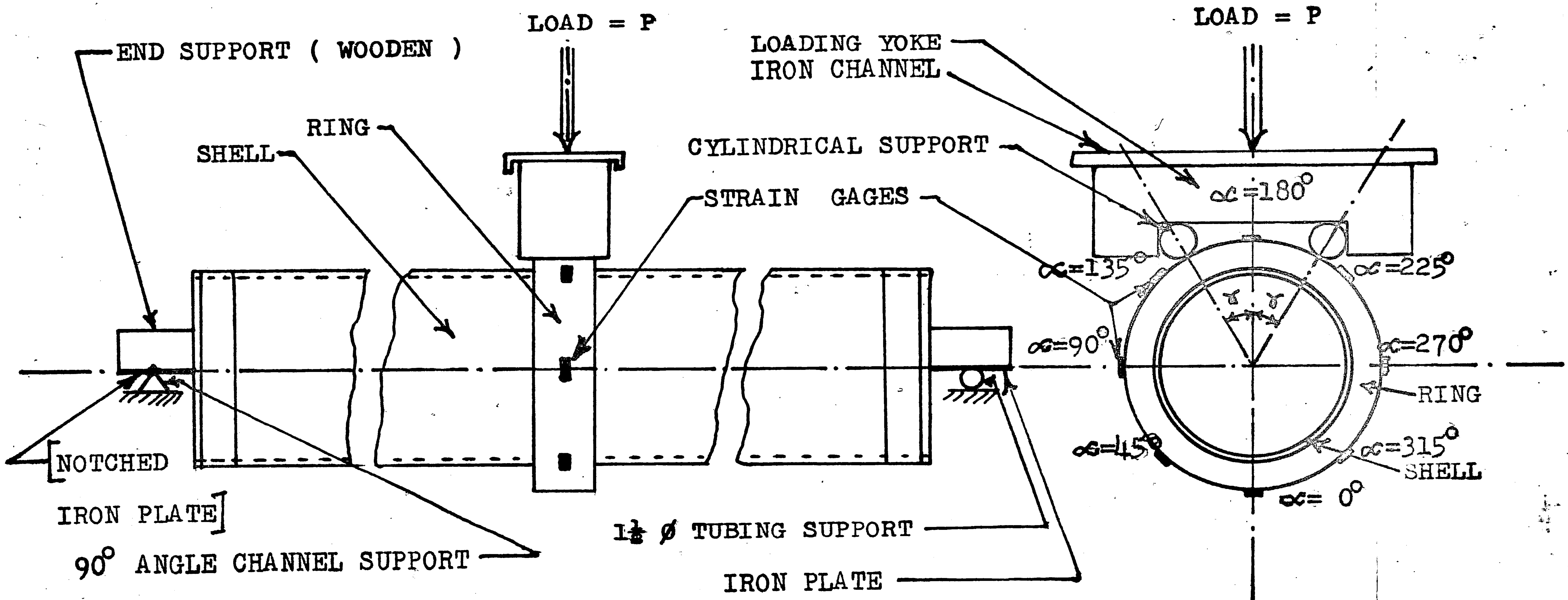

FIGURE - 1

EXPERIMENTAL SET-UP.

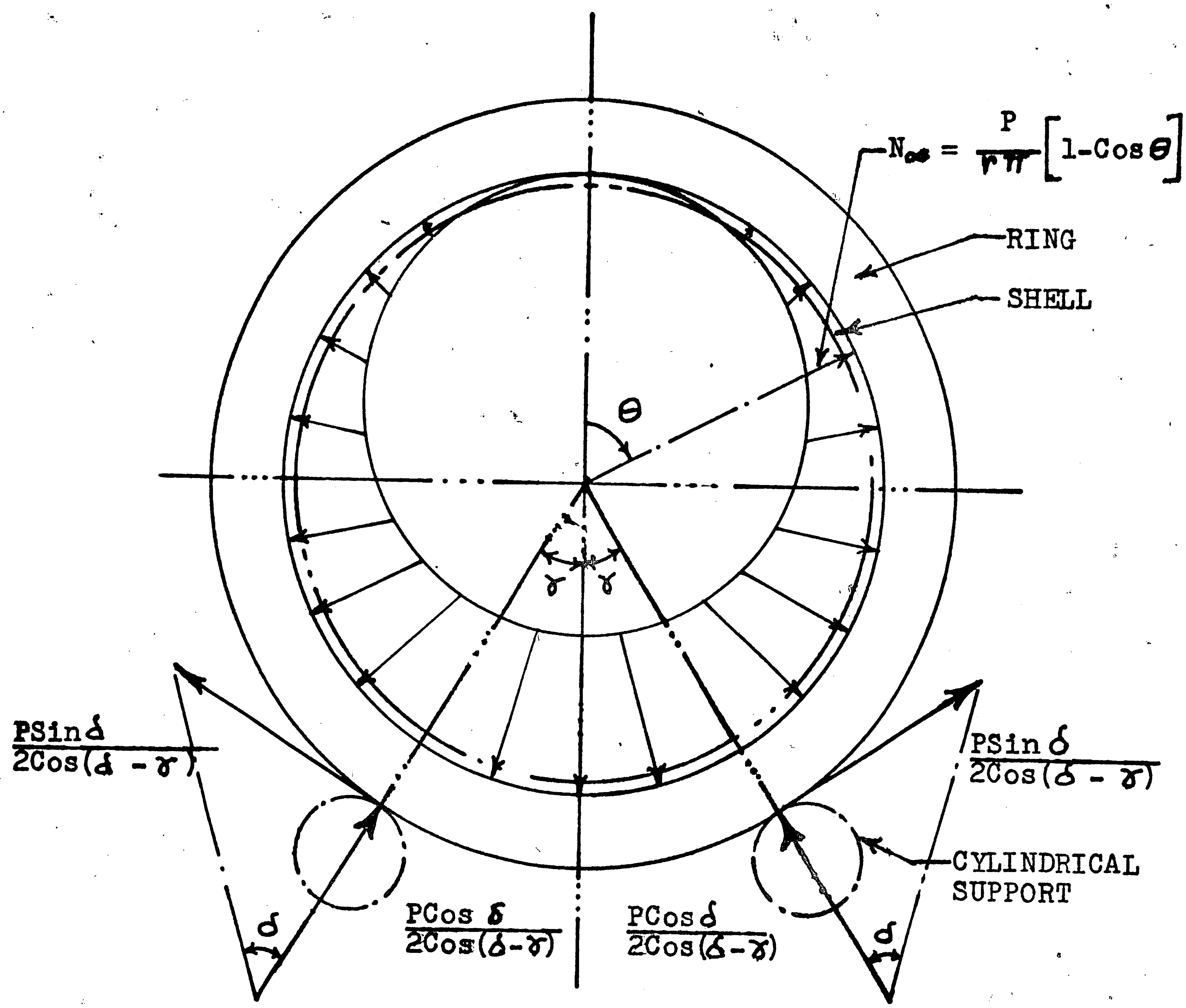

FIGURE- 2

FORCES ACTING ON THE RING.

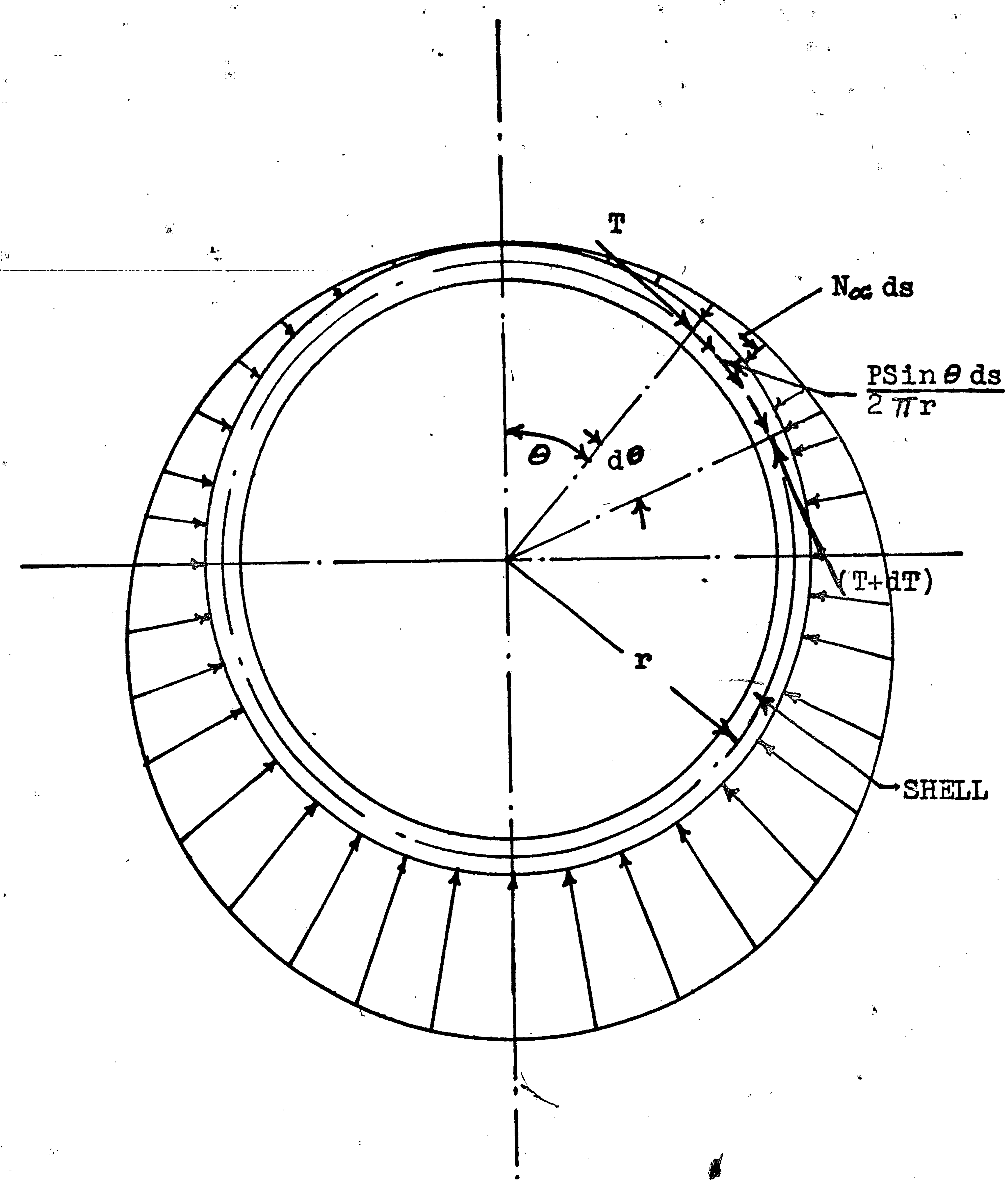

FIGURE - 3

FORCES ACTING ON AN ELEMENT OF THE SHELL

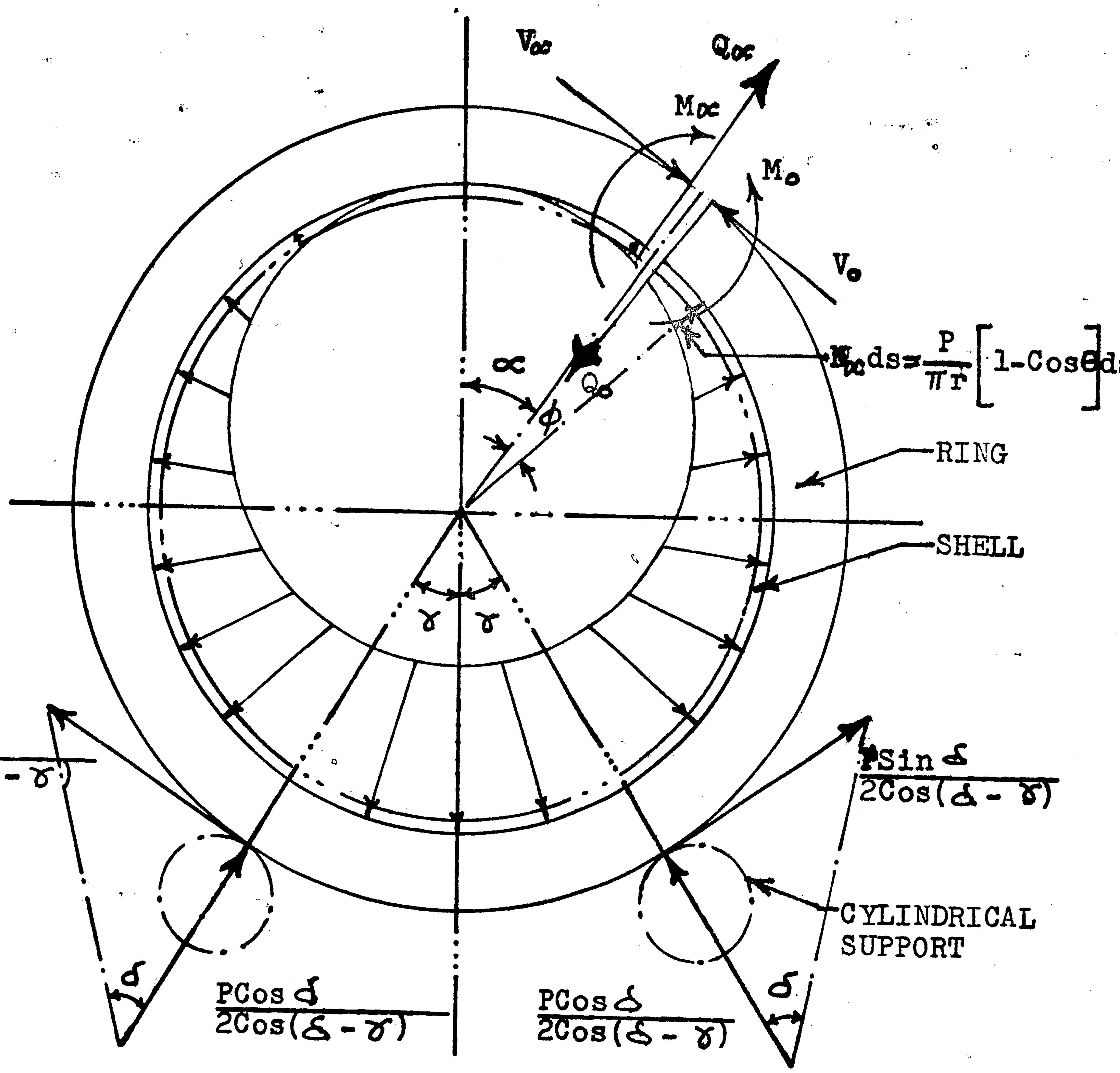

FIGURE - 4

EXTERNAL AND INTERNAL LOADS ON THE RING.

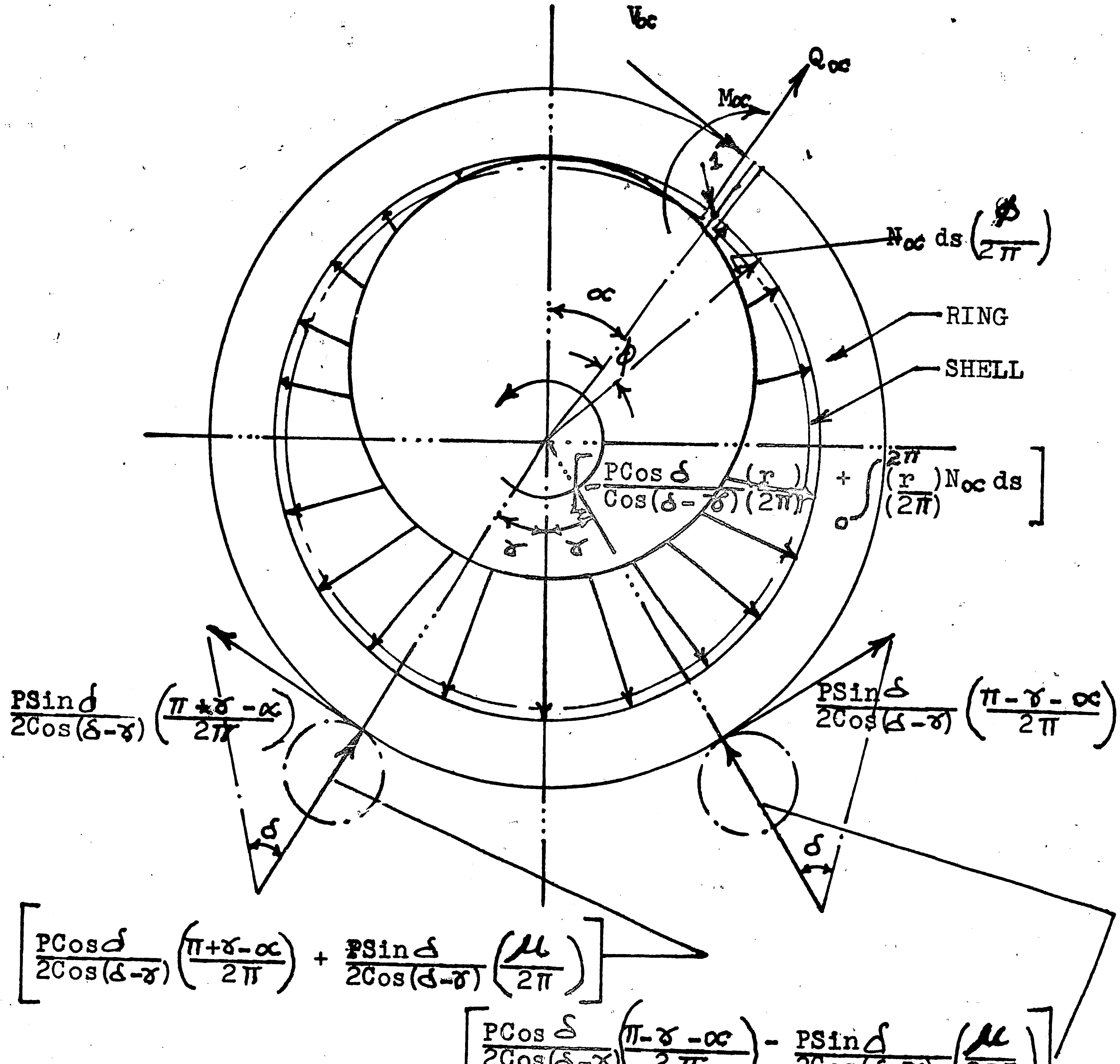

FIGURE - 5

THE "REDUCED" LOADING SYSTEM

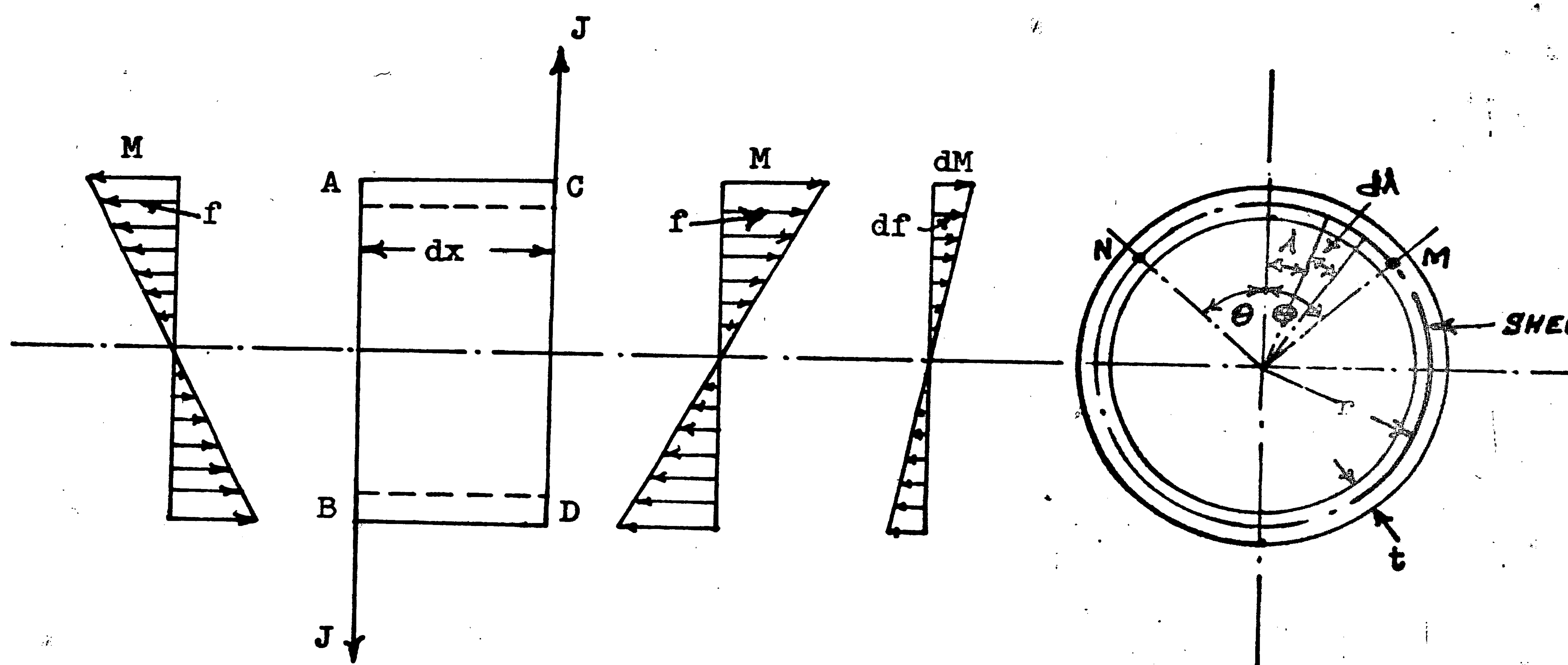

FIGURE - 6

SHEAR FORCE ON A TRANSVERSE SECTION OF THE SHELL.

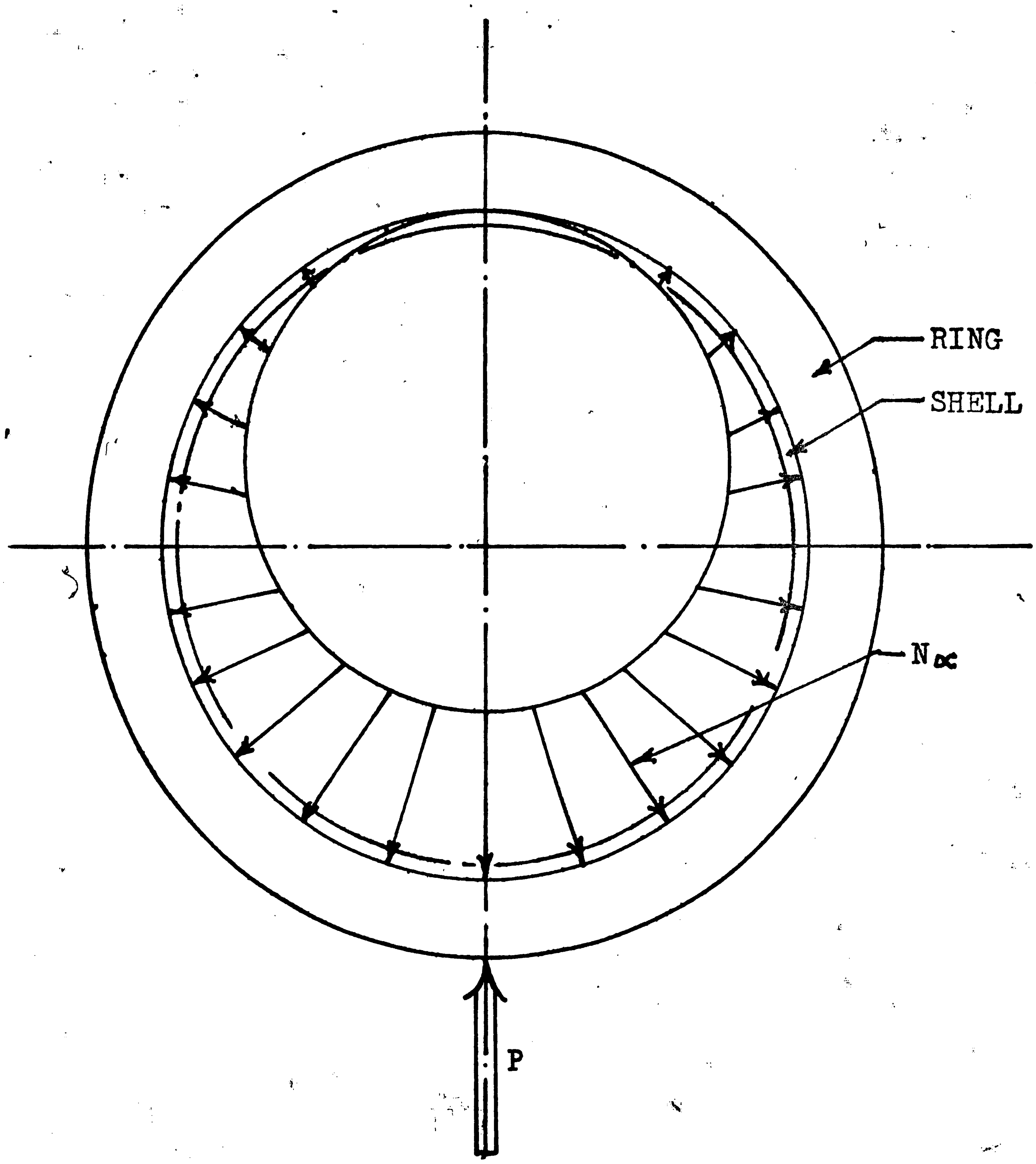

FIGURE - 7

SINGLE, CENTRALLY LOCATED REACTION.

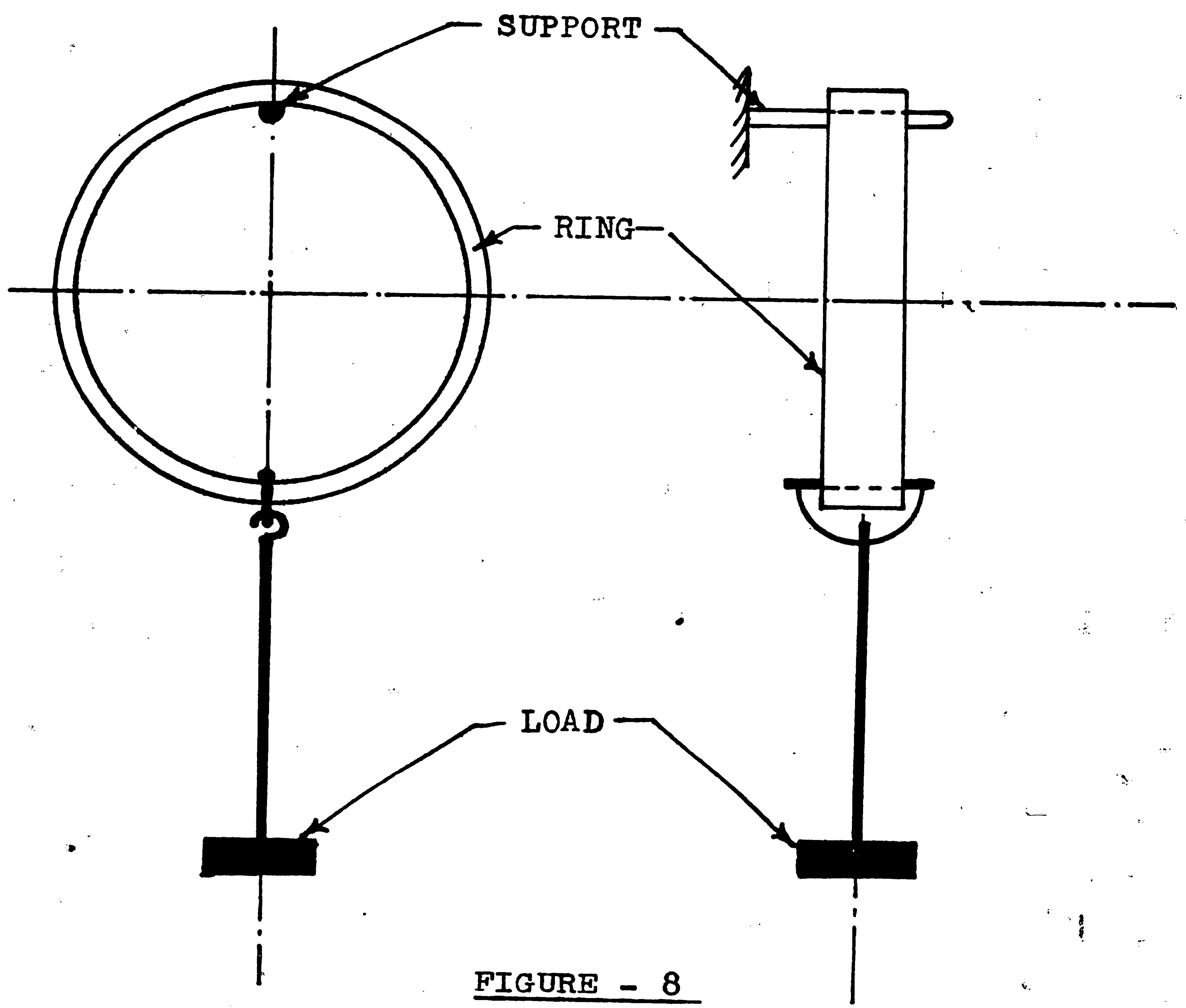

FIGURE - 8

EXPERIMENTAL SET-UP FOR MODULUS OF ELASTICITY.

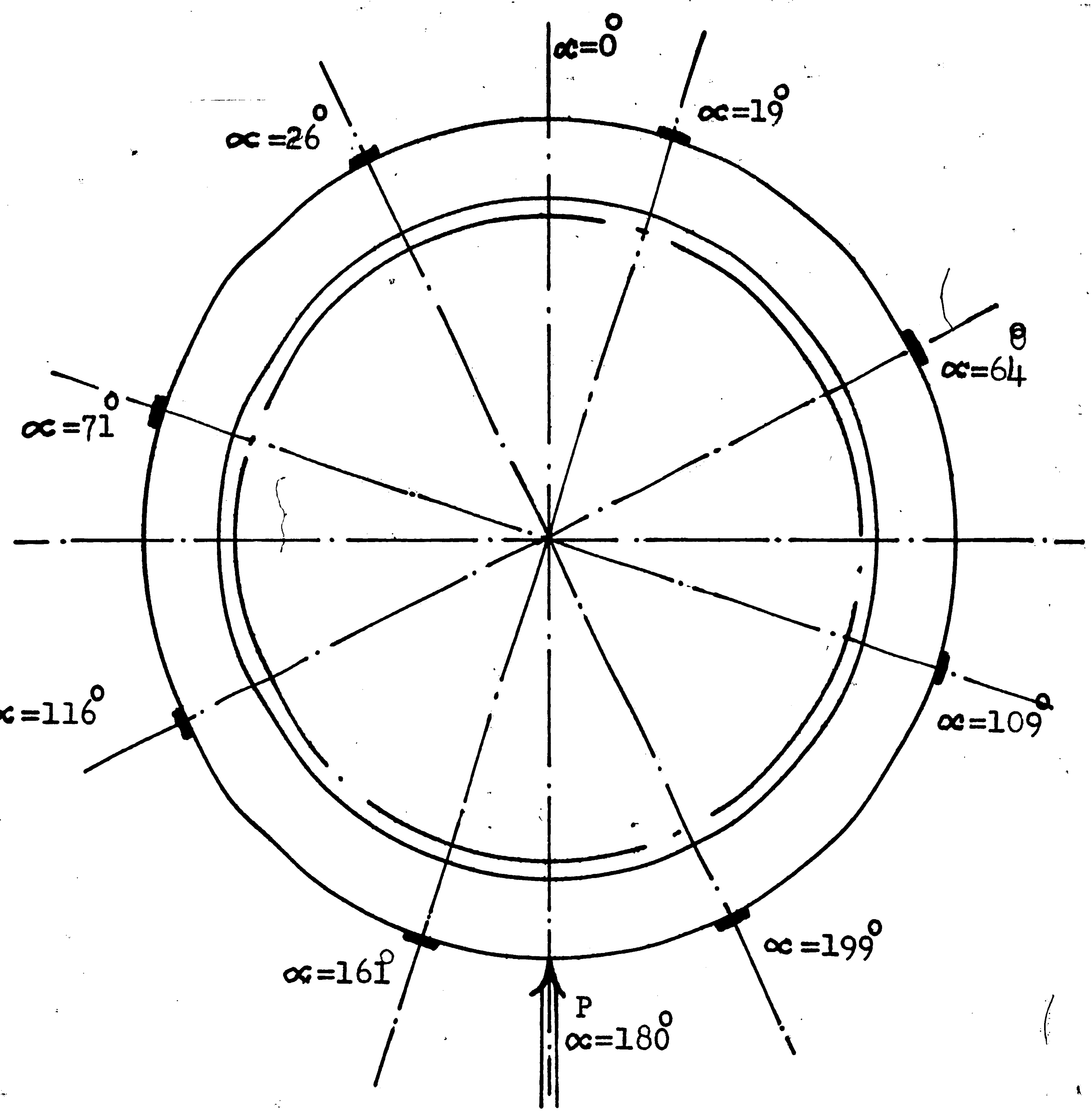

FIGURE -9

STRAIN GAGE LOCATIONS FOR SINGLE, CENTRALLY LOCATED REACTION.

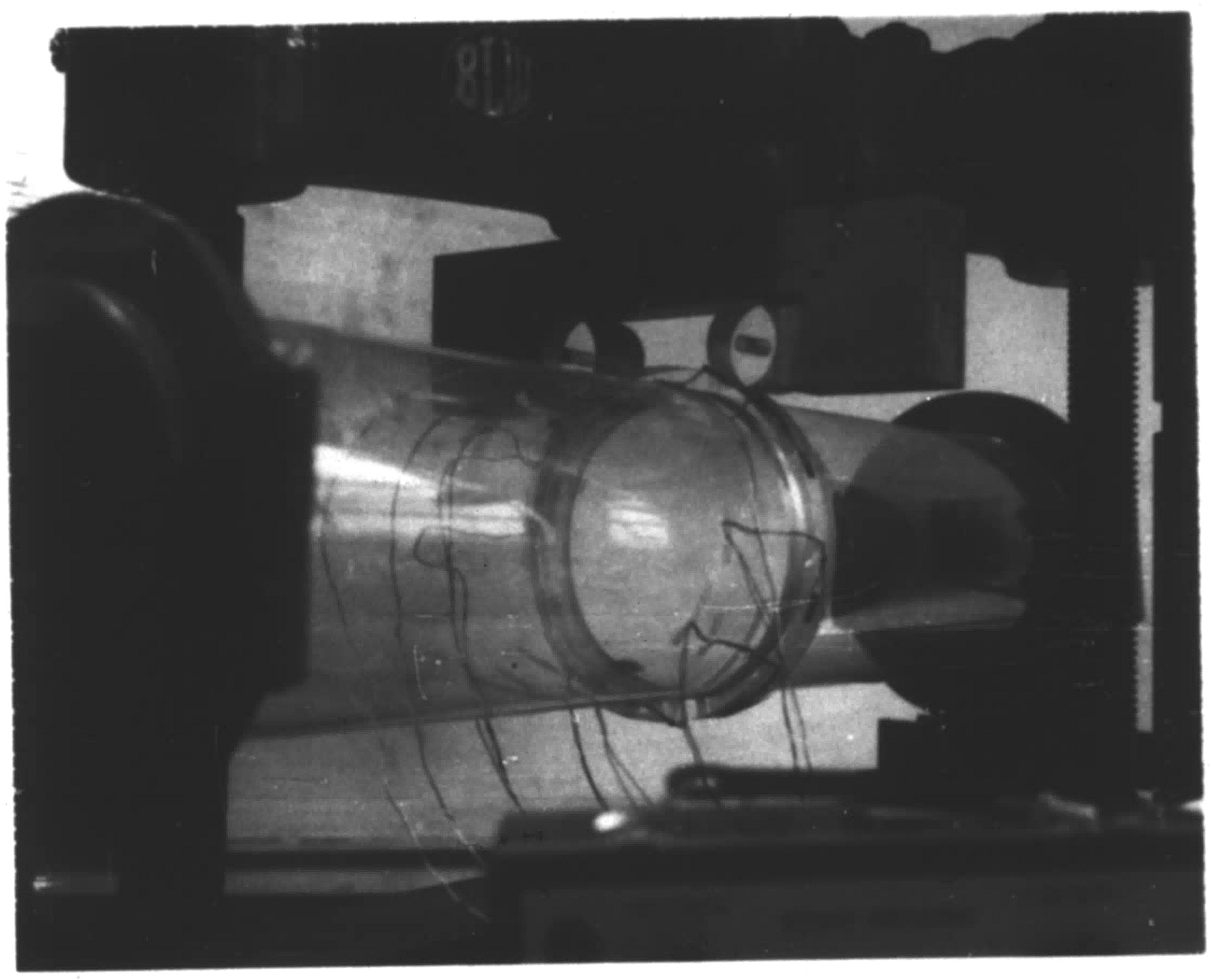

FIGURE-10

LOADING THROUGH CYLINDRICAL SUPPORTS

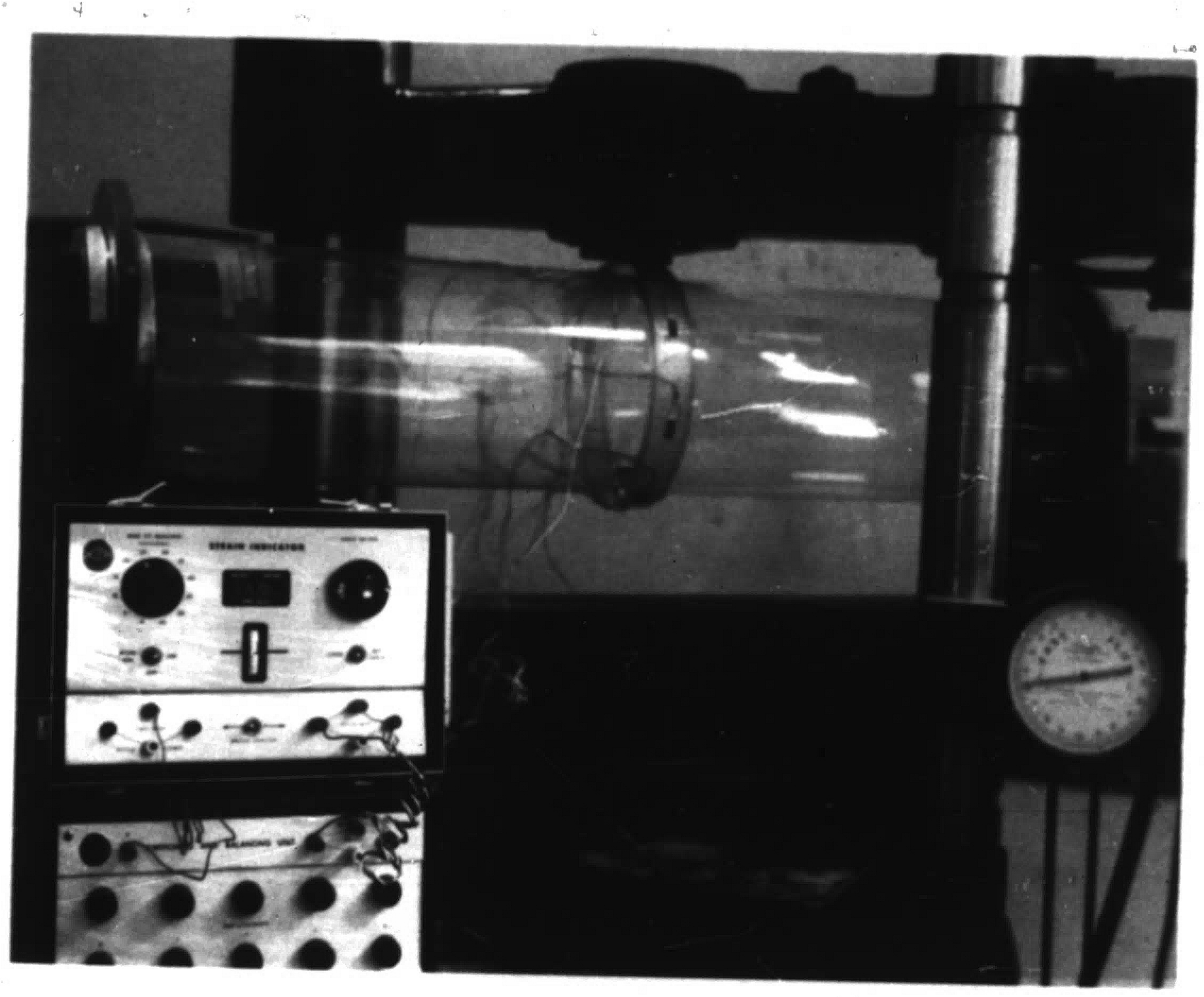

FIGURE -11

SINGLE, CENTRALLY LOCATED LOAD.

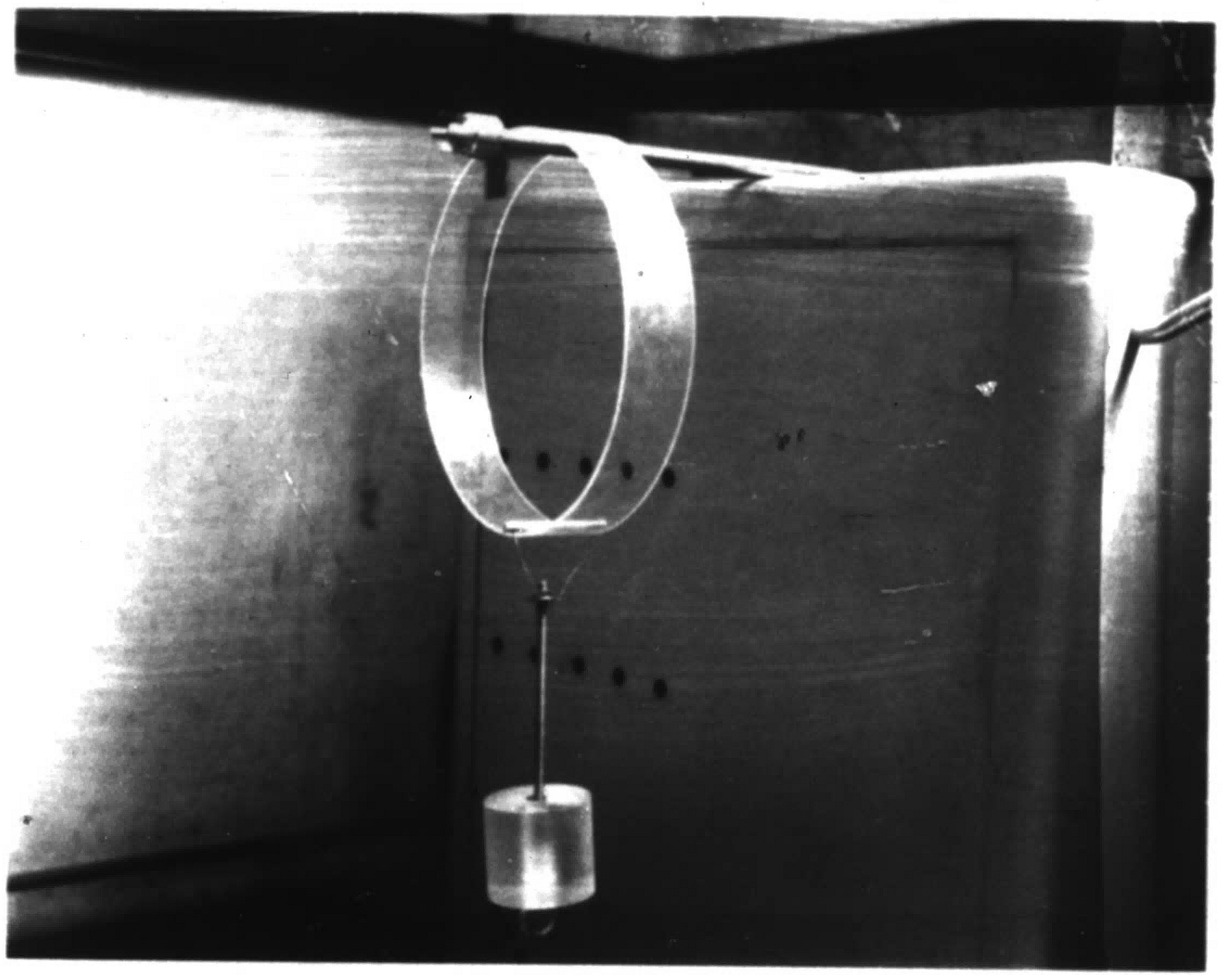

FIGURE - 12

EXPERIMENTAL SET-UP FOR DETERMINING MODULUS OF ELASTICITY.

FIGURE - 13

EXPERIMENTAL SET-UP.

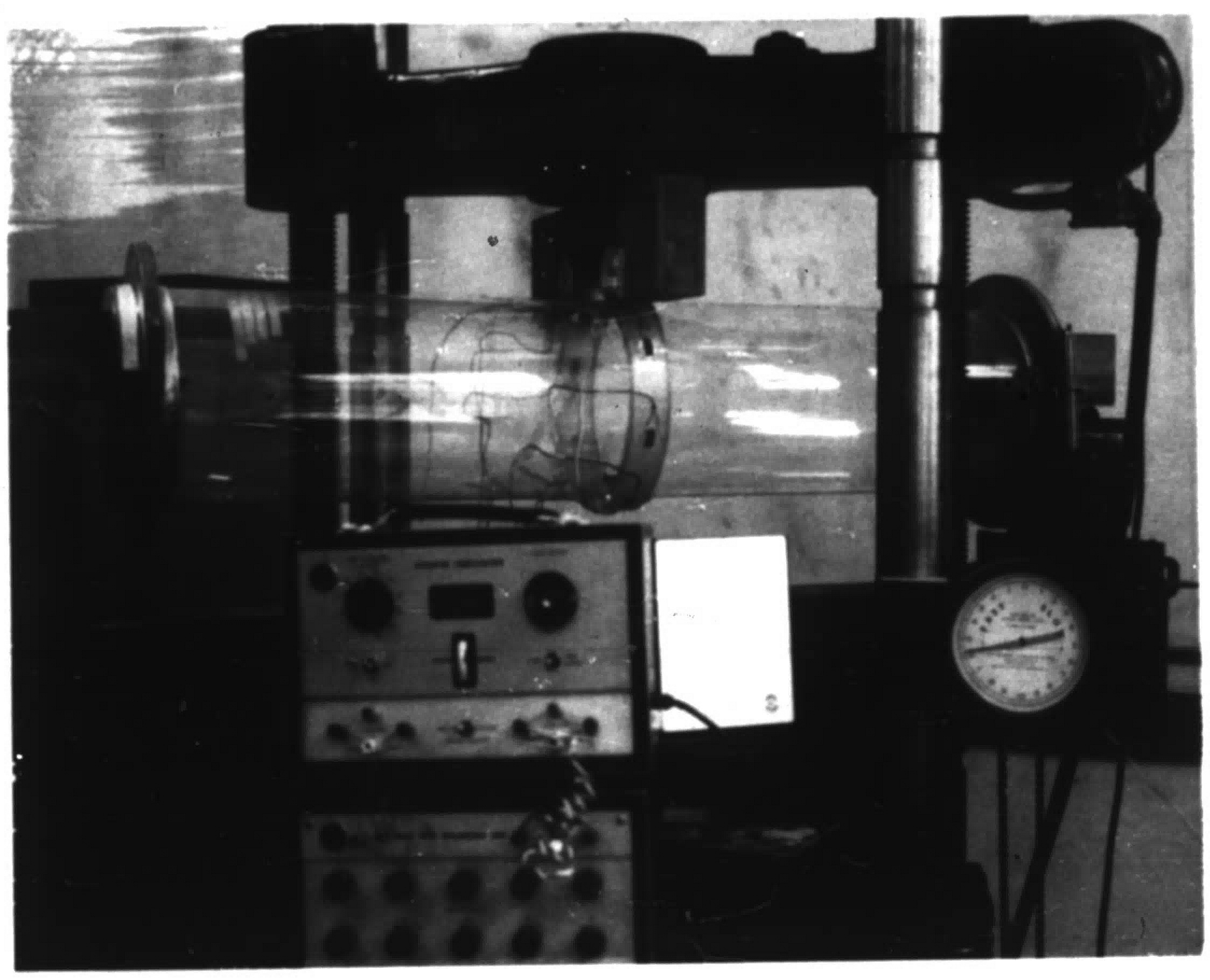

FIGURE - 14

METHOD OF SUPPORTING SHELL

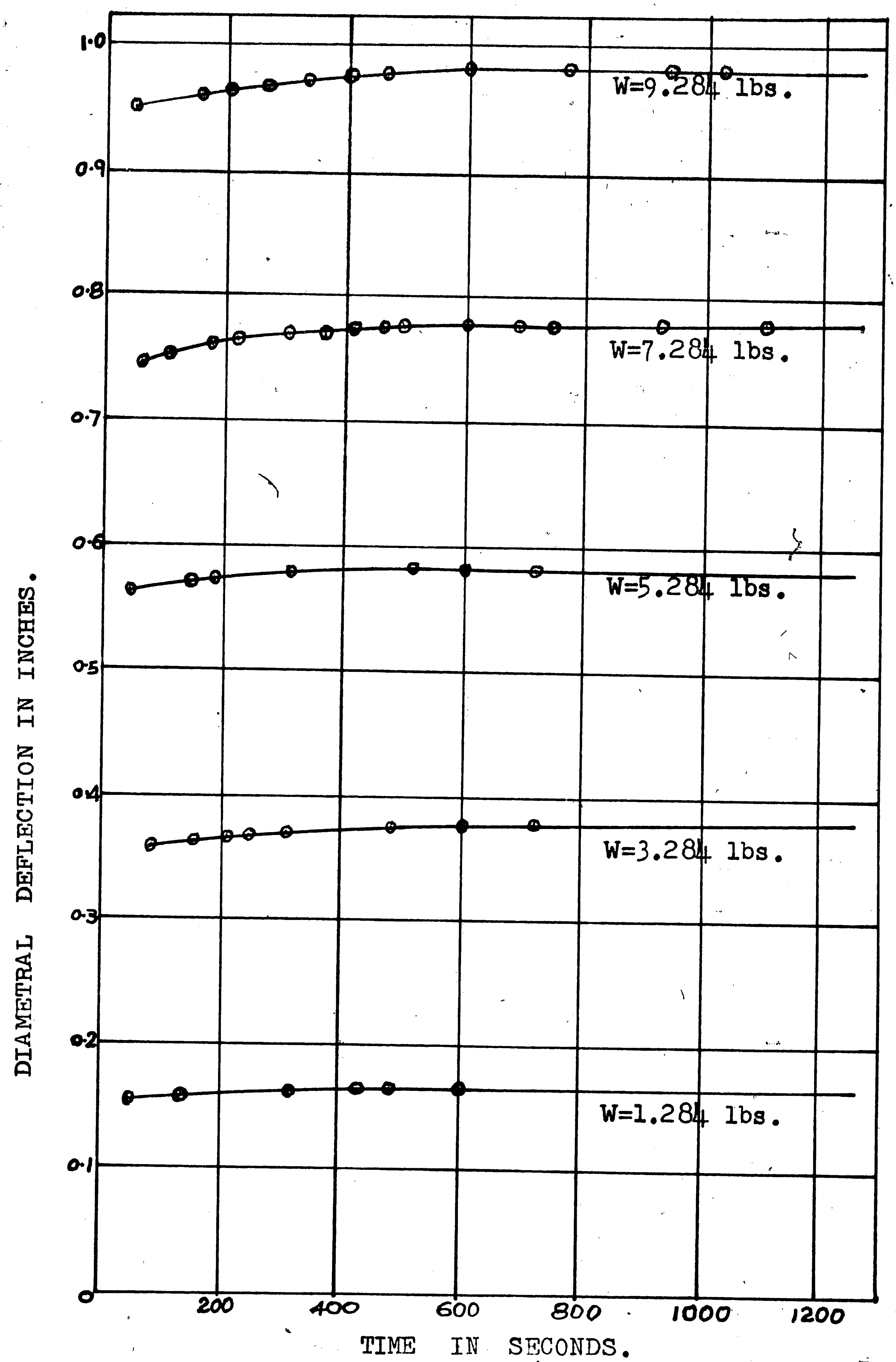

FIGURE -15

TIME-DEFLECTION CURVES FOR PLEXIGLAS PLASTIC RING.

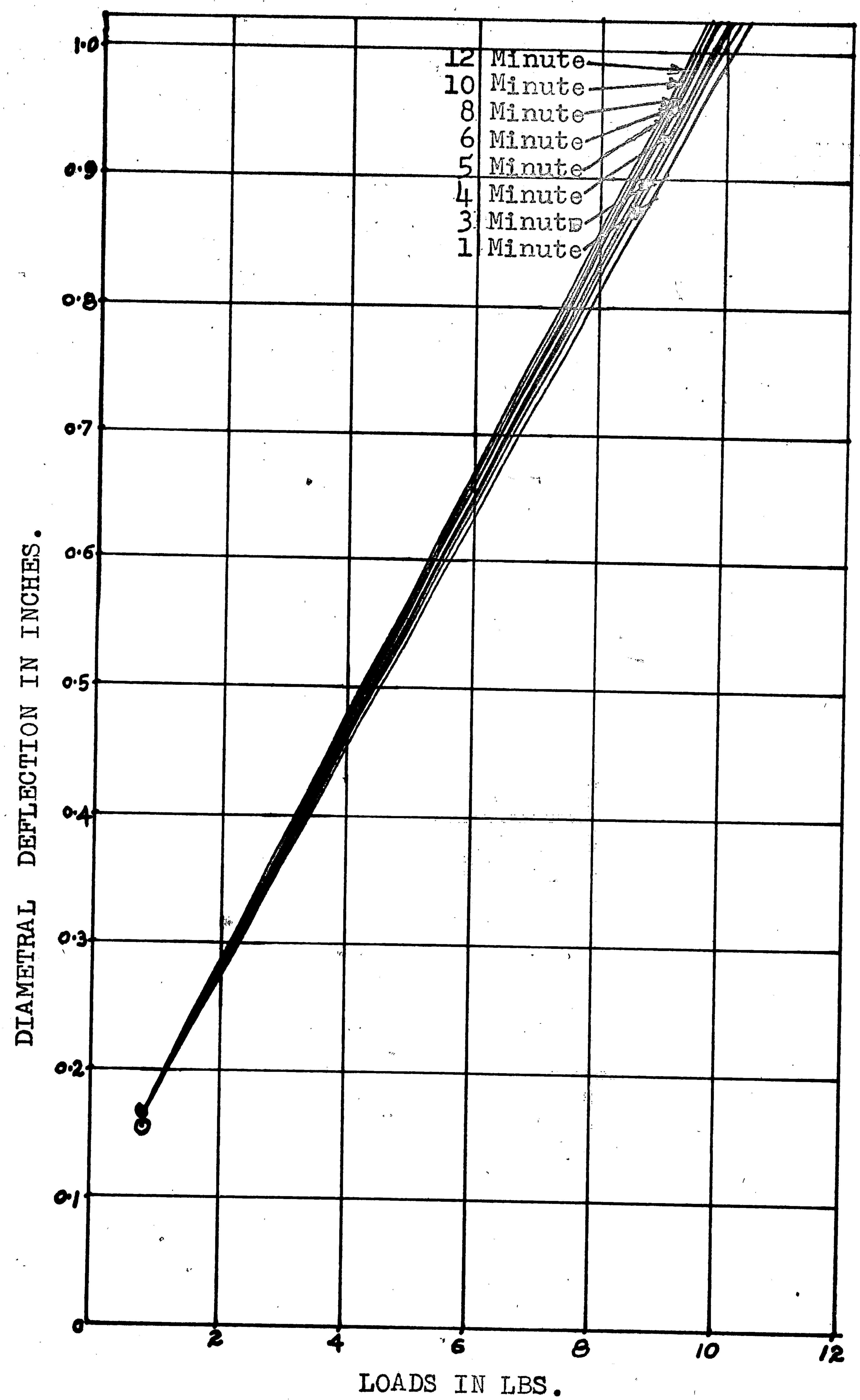

FIGURE - 16

LOAD-DEFLECTION CURVES DRAWN FROM FIGURE 15 FOR MODULUS OF ELASTICITY.

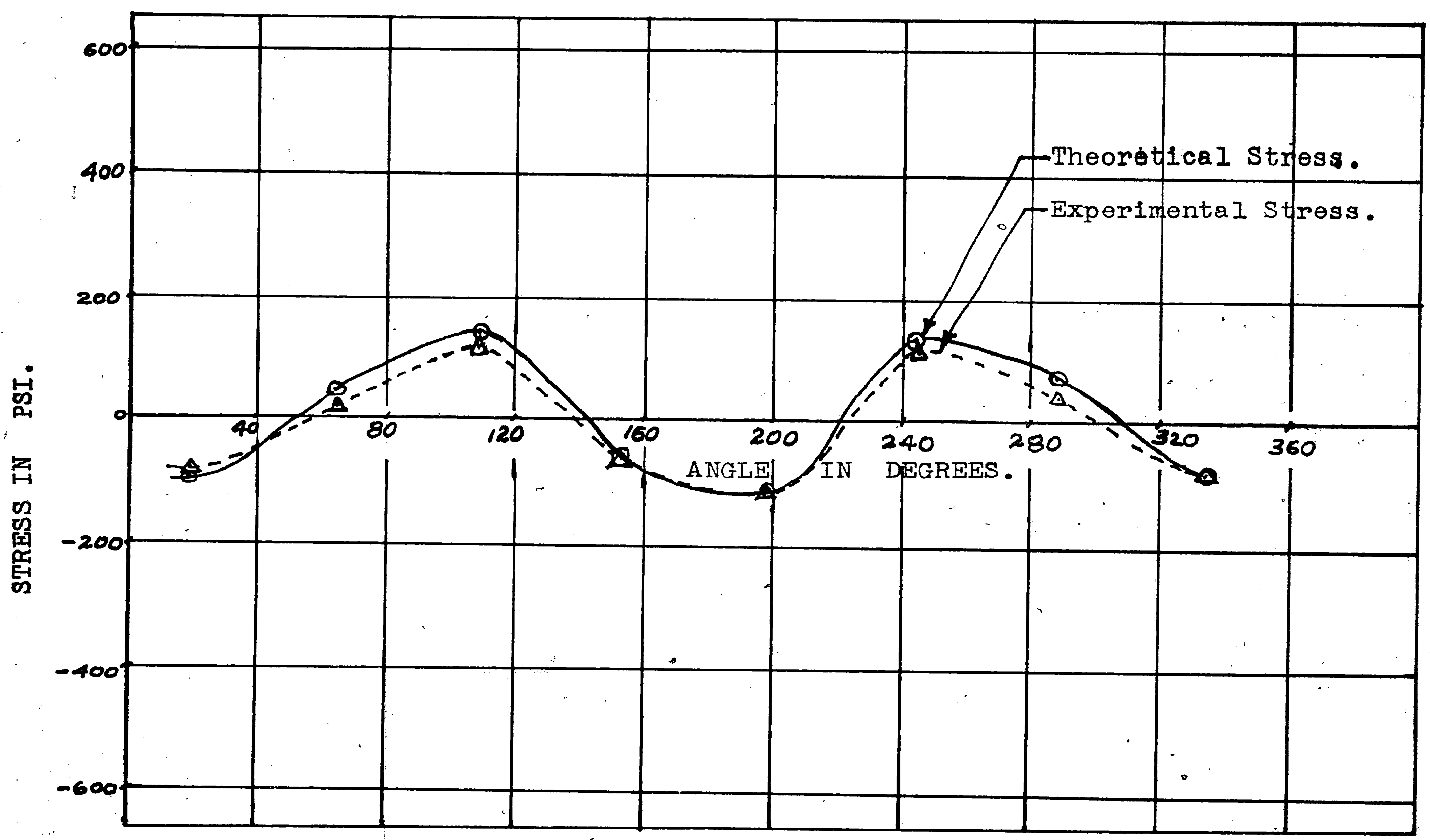

FIGURE - 17

STRESS CURVES FOR SINGLE, CENTRALLY LOCATED REACTION OF 52 LBS.

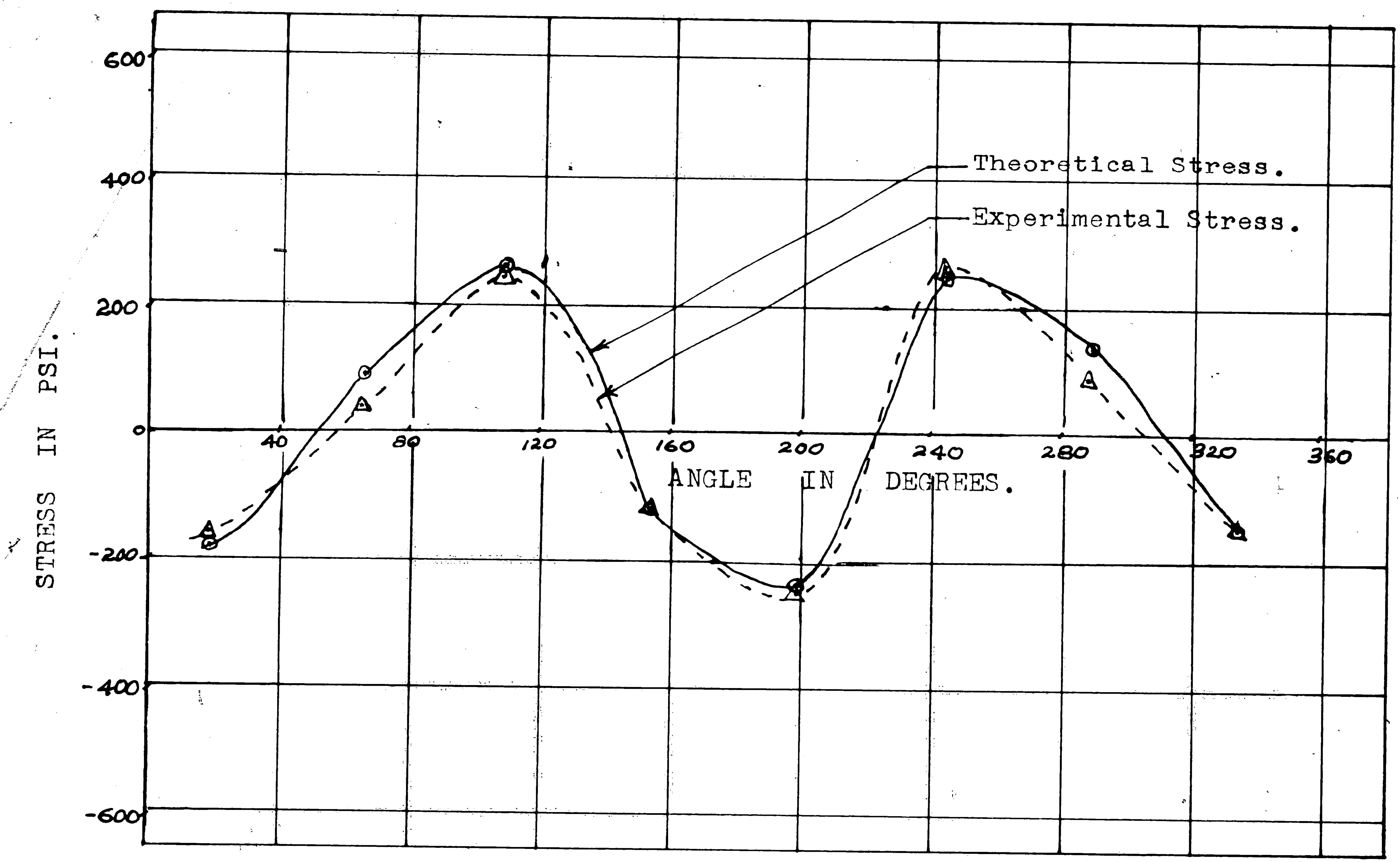

FIGURE - 18

STRESS CURVES FOR SINGLE, CENTRALLY LOCATED REACTION OF 101 LBS.

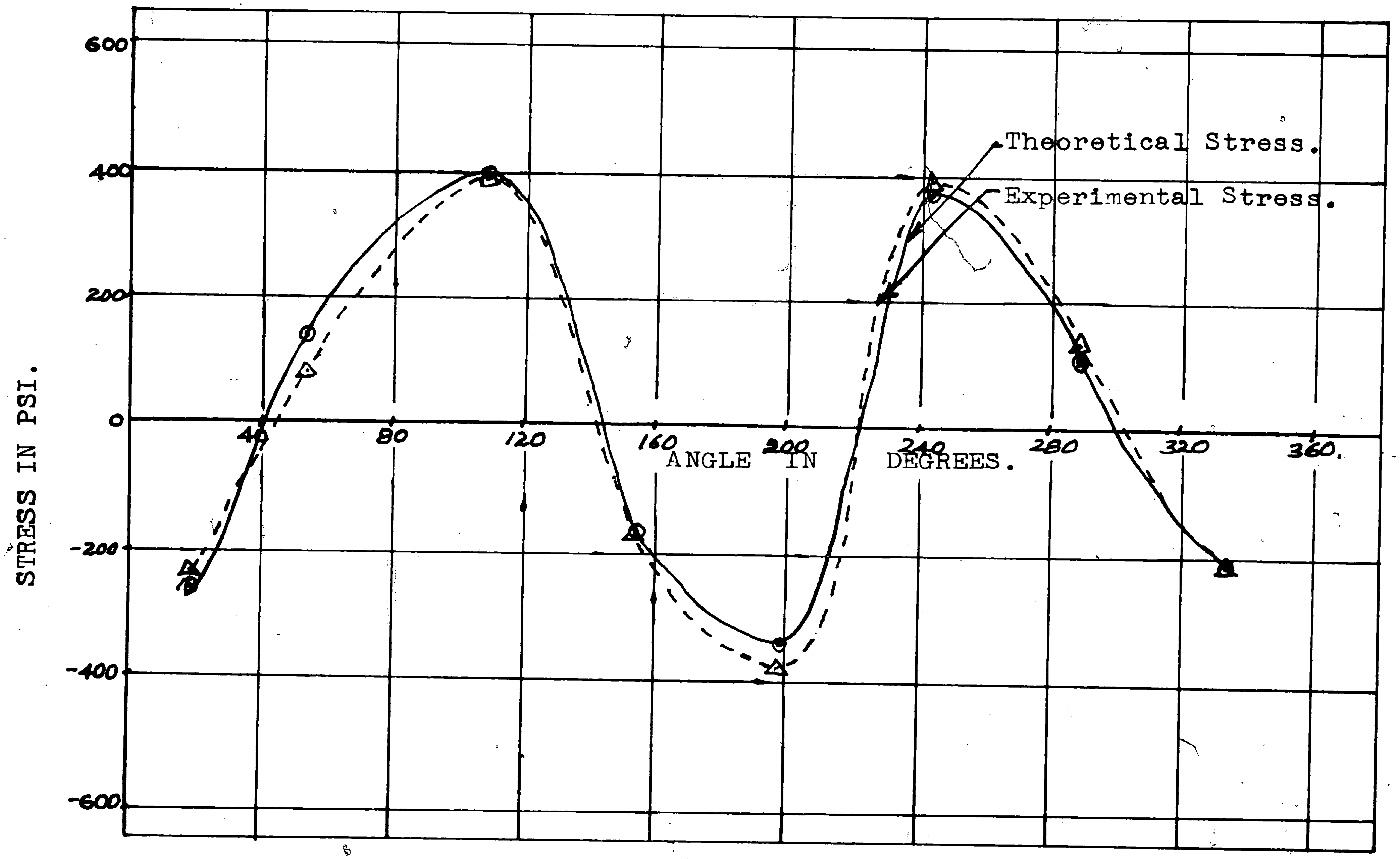

FIGURE - 19

STRESS CURVES FOR SINGLE, CENTRALLY LOCATED REACTION OF 151 LBS.

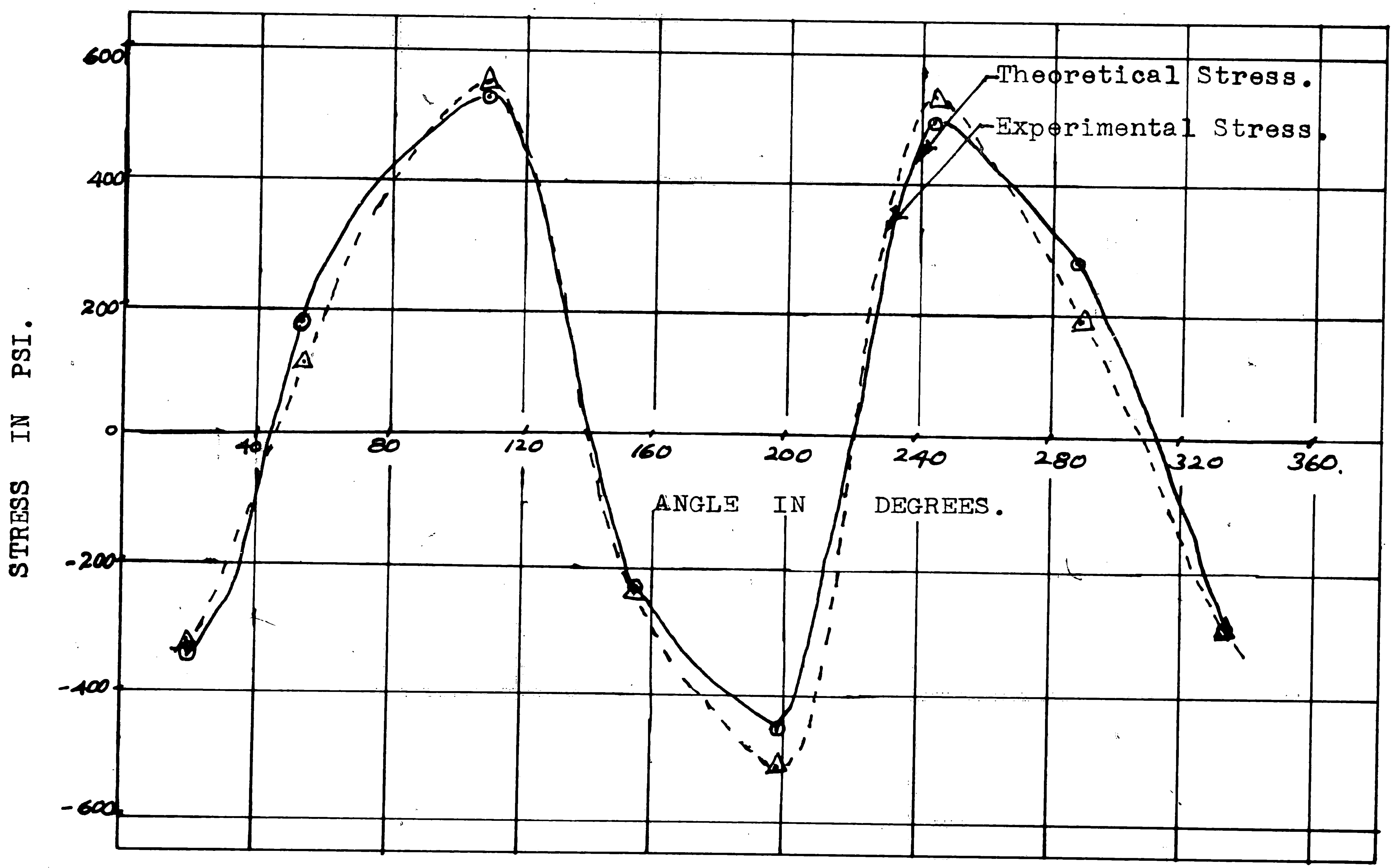

FIGURE - 20

STRESS CURVES FOR SINGLE, CENTRALLY LOCATED REACTION OF 200 LBS.

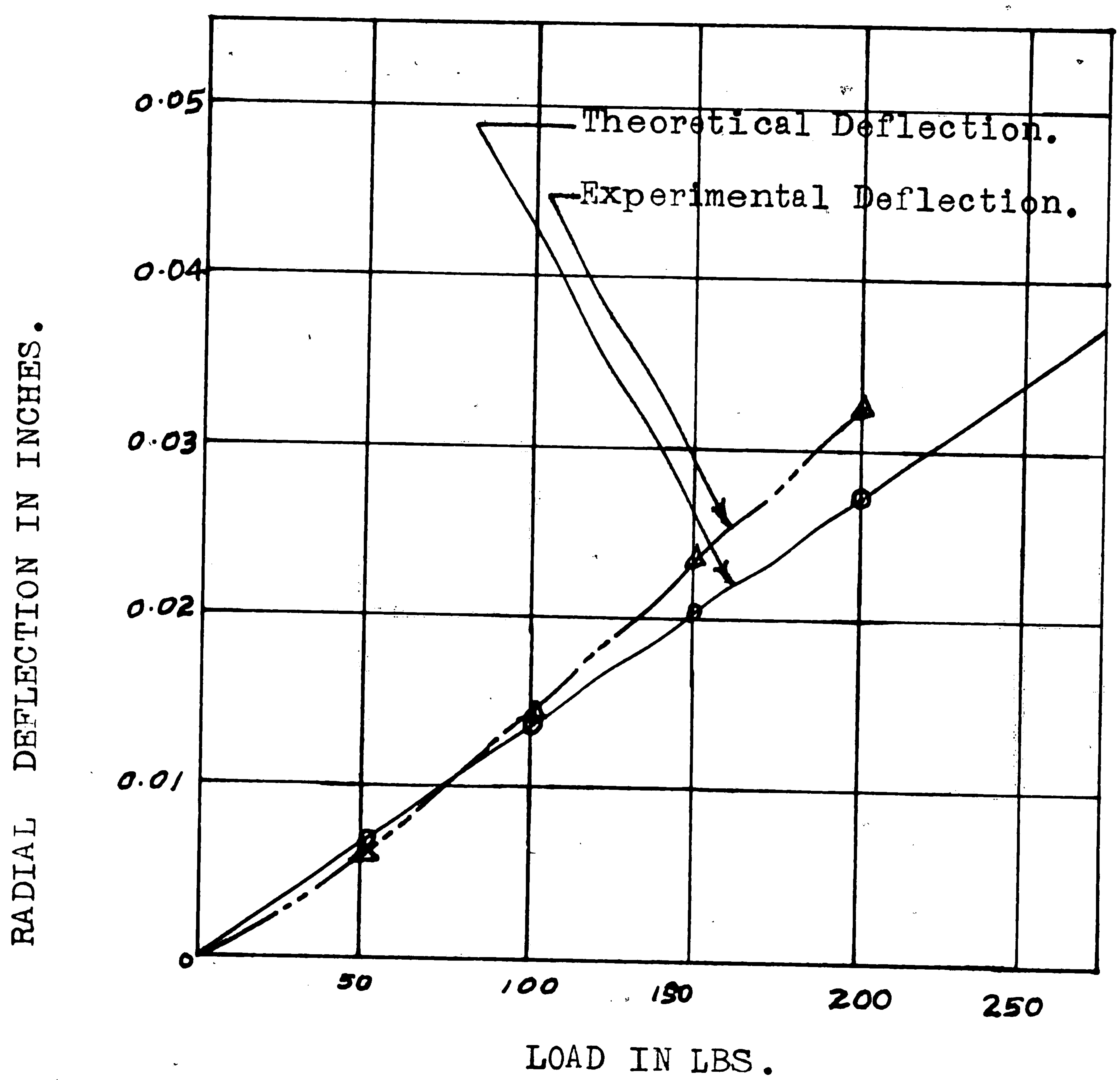

FIGURE - 21

LOAD-DEFLECTION CURVES FOR SINGLE, CENTRALLY LOCATED REACTION.

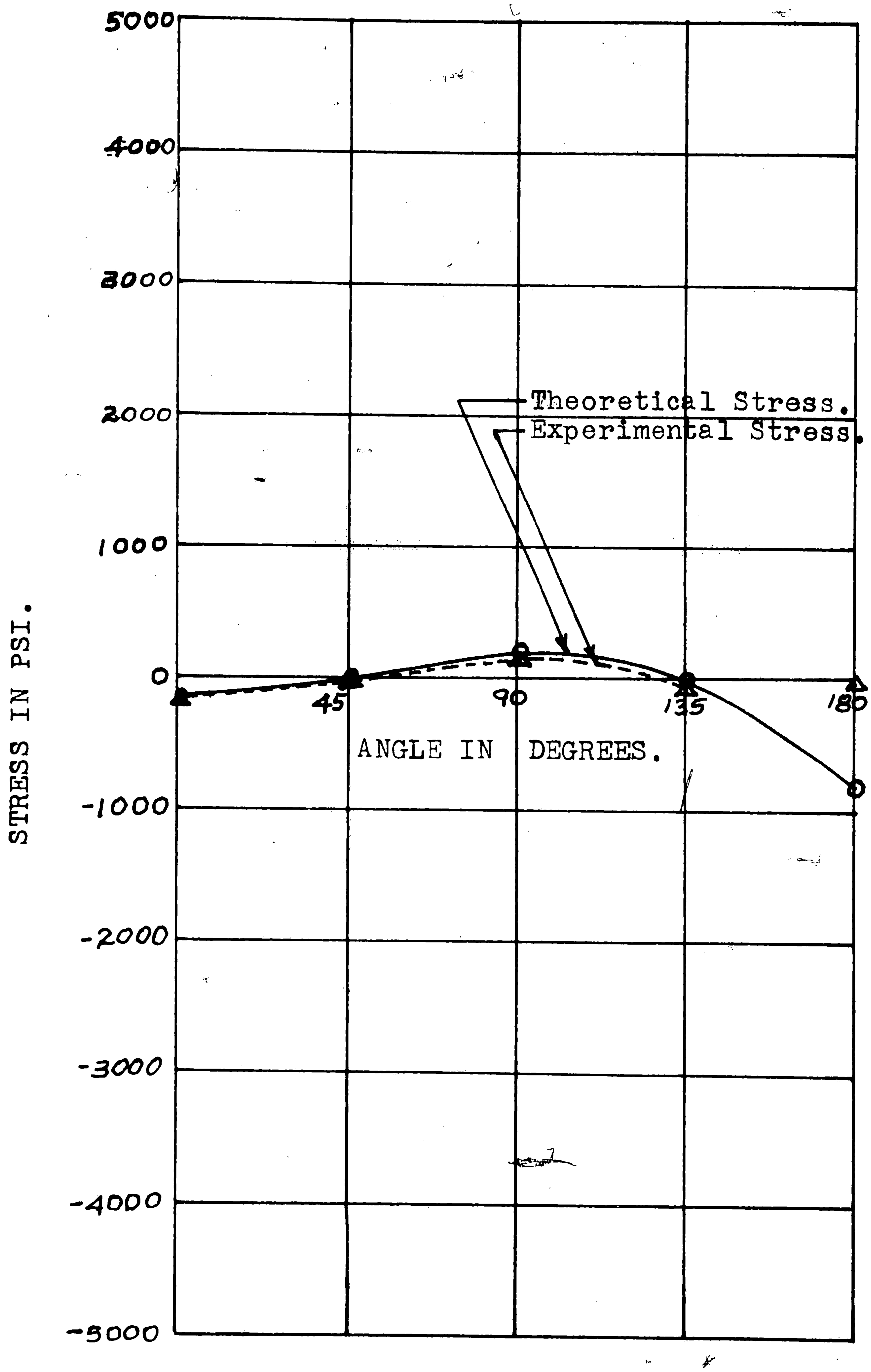

FIGURE - 22

STRESS CURVES FOR ONLY NORMAL REACTION LOCATED 30° FROM THE VERTICAL. TOTAL LOAD=100 LBS.

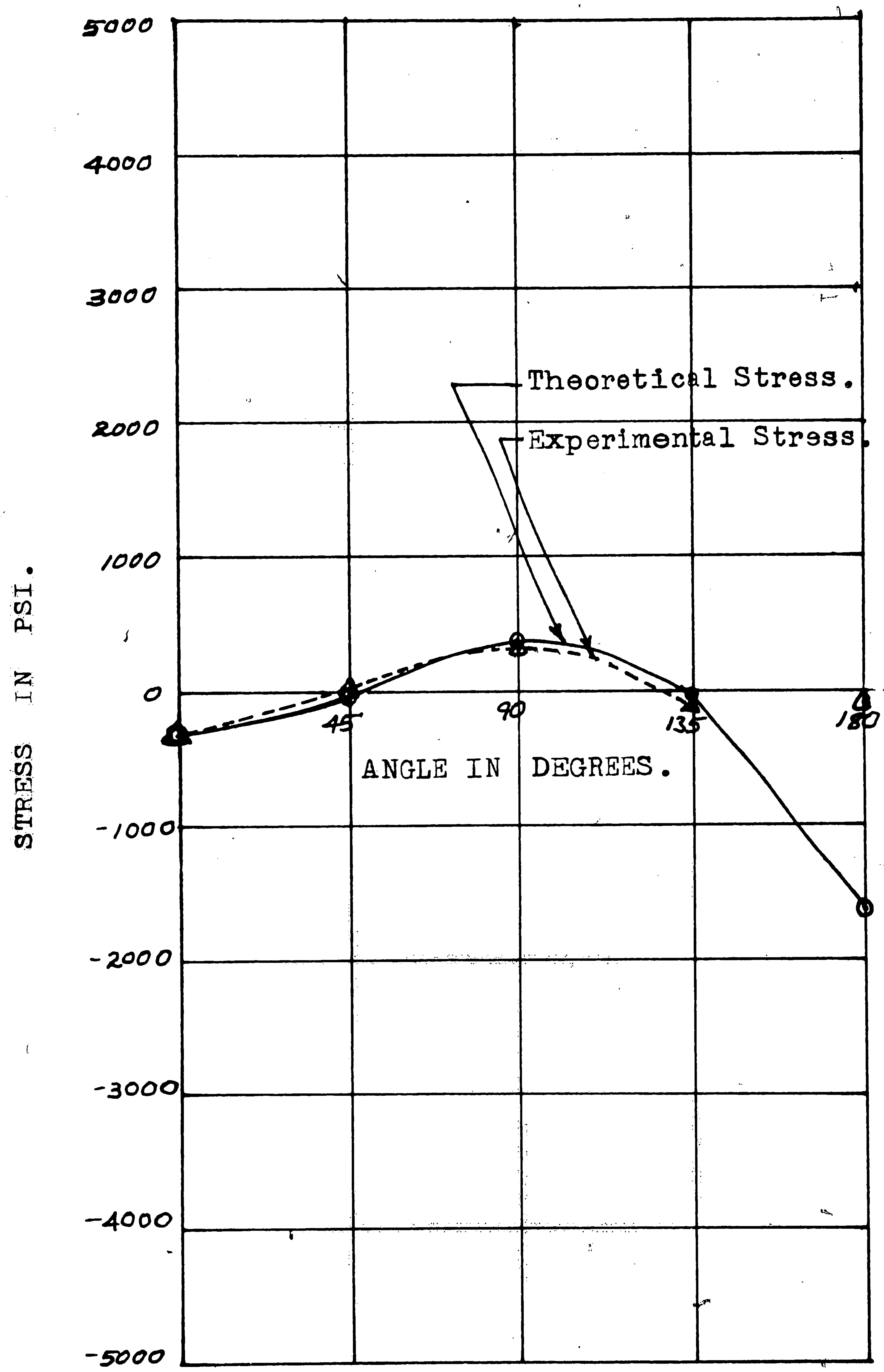

FIGURE -23

STRESS CURVES FOR ONLY NORMAL REACTION LOCATED 30° FROM THE VERTICAL. TOTAL LOAD = 200 LBS.

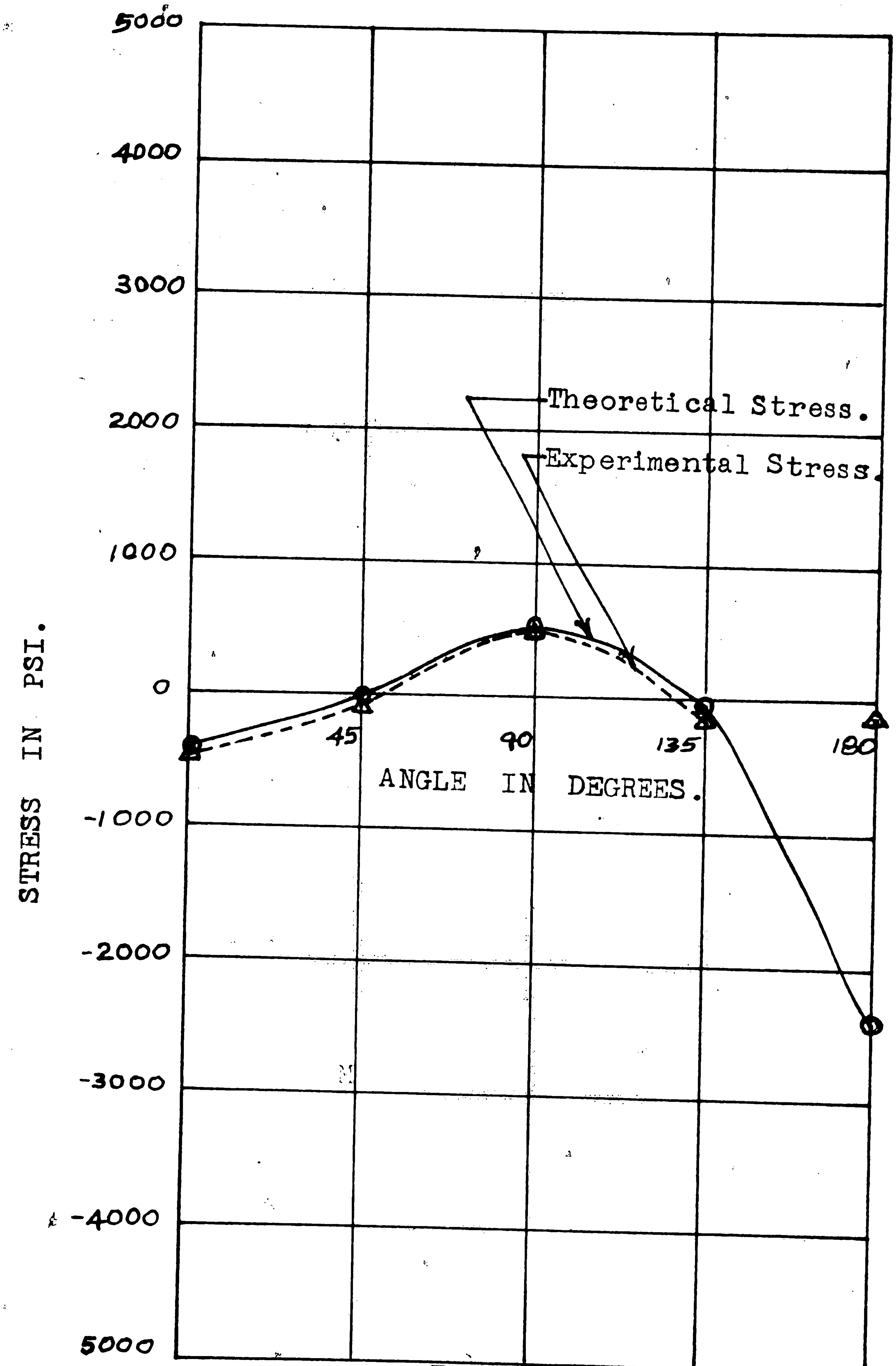

FIGURE -24

STRESS CURVES FOR ONLY NORMAL REACTION LOCATED 30° FROM THE VERTICAL. TOTAL LOAD = 300 LBS.

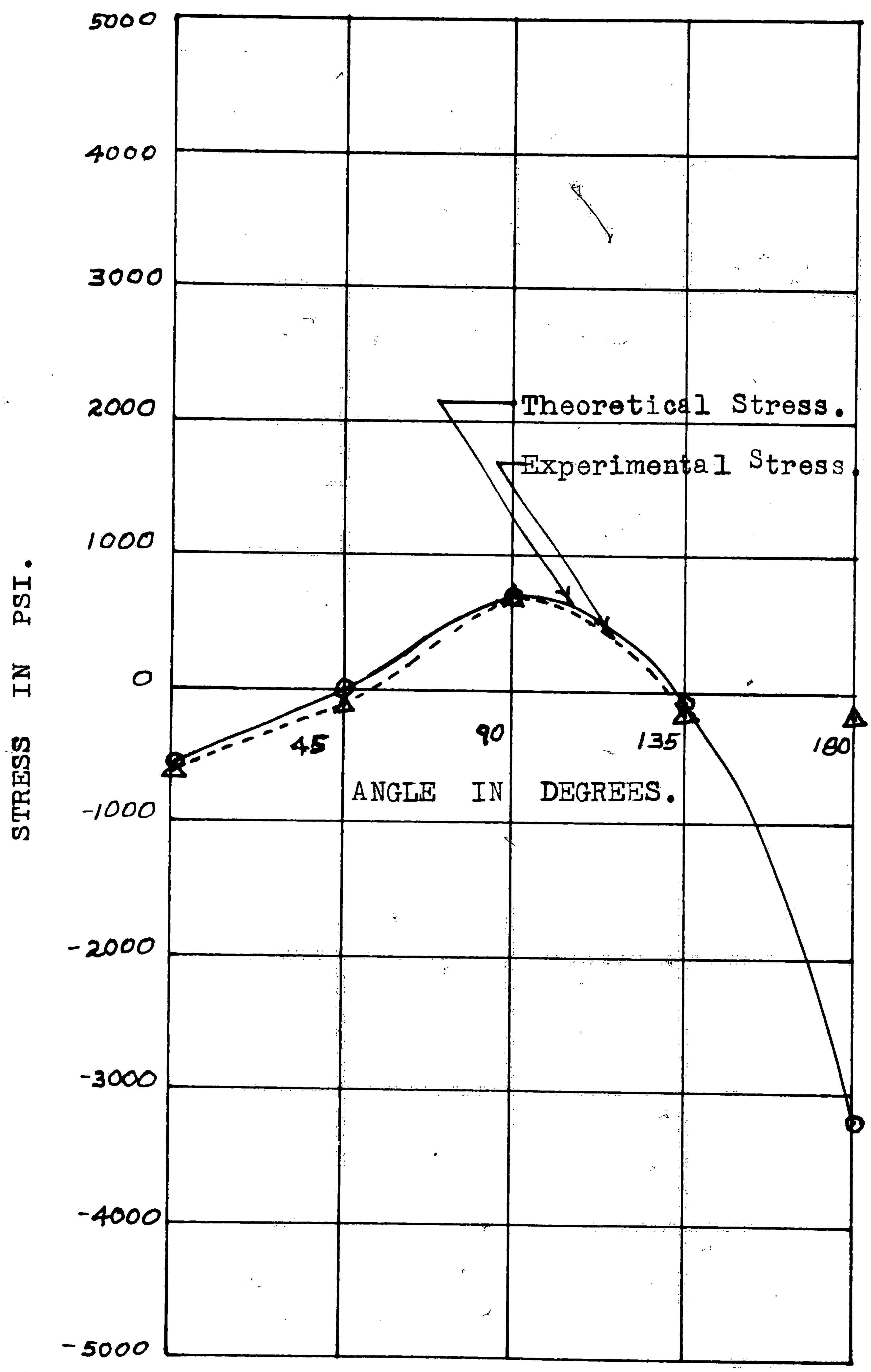

FIGURE - 25

STRESS CURVES FOR ONLY NORMAL REACTION LOCATED 30° FROM THE VERTICAL. TOTAL LOAD = 400 LBS.

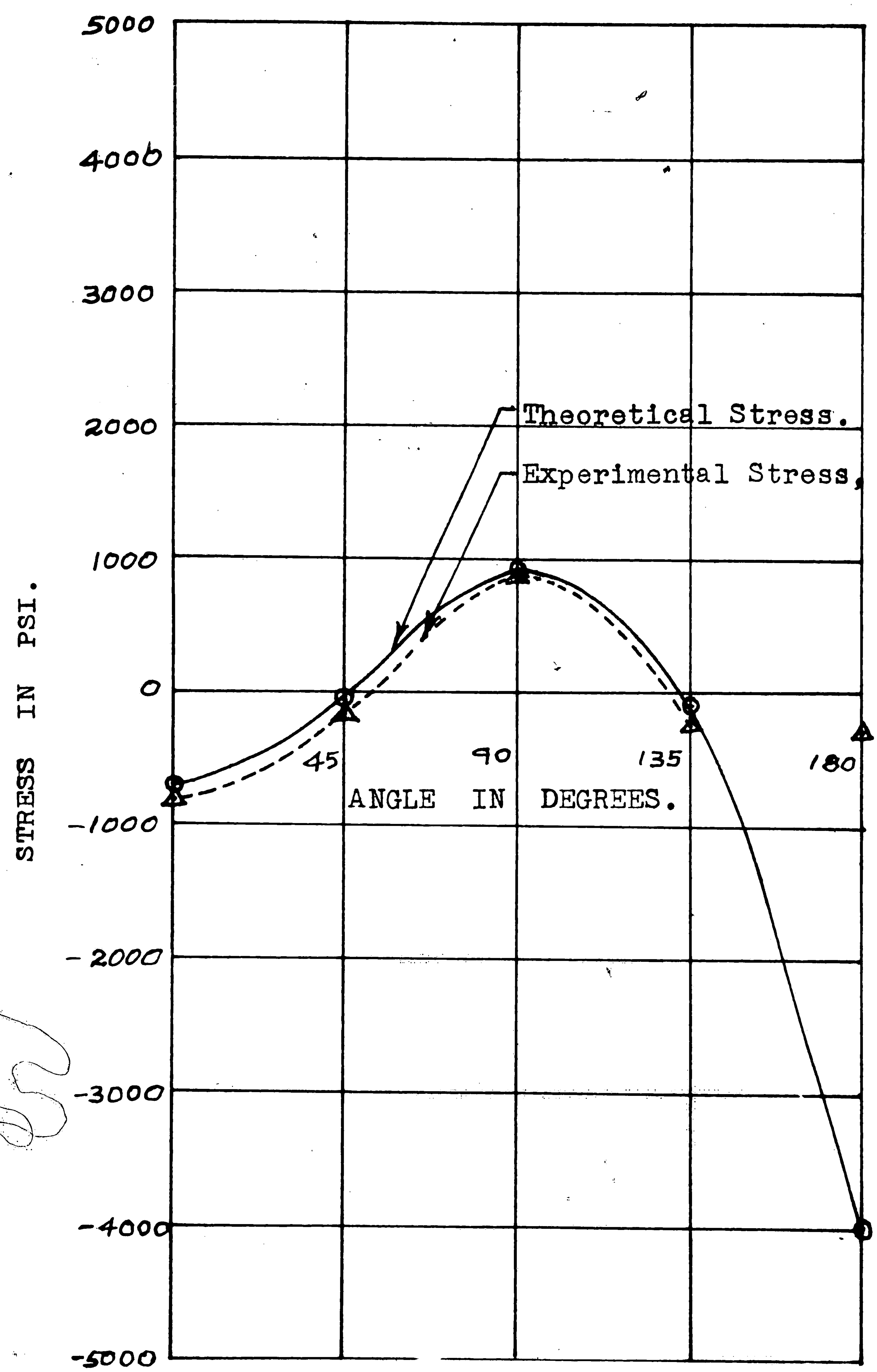

FIGURE - 26

STRESS CURVES FOR ONLY NORMAL REACTION LOCATED 30° FROM THE VERTICAL. TOTAL LOAD = 500 LBS.

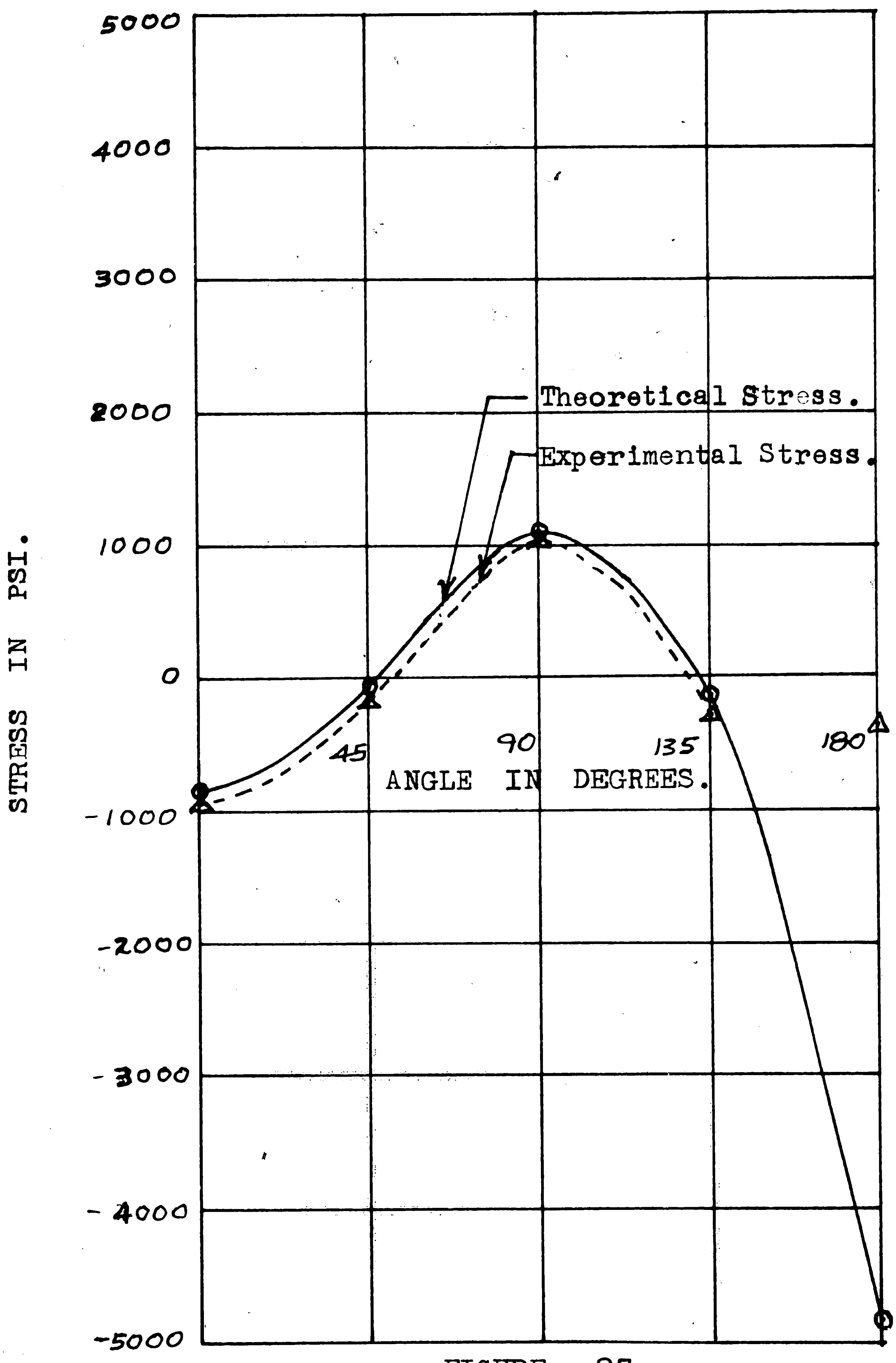

FIGURE - 27

STRESS CURVES FOR ONLY NORMAL REACTION LOCATED 30° FROM THE VERTICAL. TOTAL LOAD = 600 LBS.

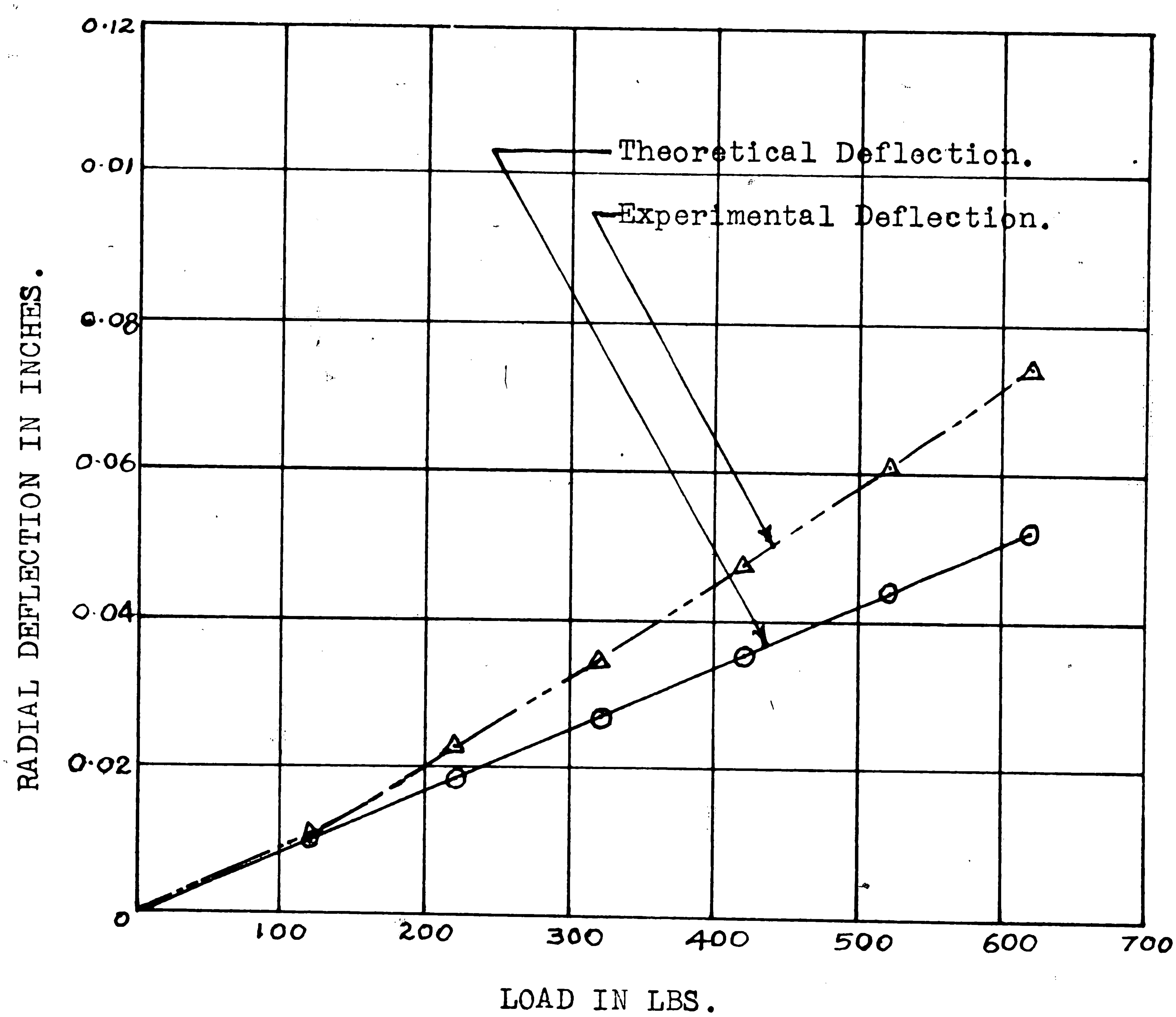

FIGURE - 28.

LOAD-DEFLECTION CURVES FOR ONLY NORMAL REACTION LOCATED 30° FROM THE VERTICAL.

TABLE - I

THEORETICAL AND EXPERIMENTAL STRESSES FOR THE SINGLE, CENTRALLY LOCATED REACTION.

ANGLE IN DEGREES		19	64	109	154	-161	-116	-71	-26
LOAD IN LBS.	STRESS IN PSI.								
52	THEORETICAL	-90.2	+48	+137	-58.2	-112	+129	+72.5	-75.5
	EXPERIMENTAL	-80	+21.76	+119	-60.2	-111	+119.5	+40.5	-74
101	THEORETICAL	-175	+93	+266	-113	-234	+251	+140	-146.5
	EXPERIMENTAL	-154	+49.9	+250	-111,8	-247	+266	+91	-141
151	THEORETICAL	-262	+139	+398	-169	-348	+375	+210	-219
	EXPERIMENTAL	-241	+80	+390	-175	-386	+390.4	+141	-220
200	THEORETICAL	-347	+184	+527	-224	-462	+496	+278	-290
	EXPERIMENTAL	-326	+112	+554	-237	-516	+540	+195	-296

+ Tensile Stress and - Compressive Stress.

TABLE - II

THEORETICAL AND EXPERIMENTAL DEFLECTIONS FOR THE SINGLE, CENTRALLY LOCATED REACTION

LOAD IN LBS.	51	100.5	150.5	200
THEORETICAL DEFLECTION IN INCHES.	0.00698	0.01376	0.02060	0.02736
EXPERIMENTAL DEFLECTION IN INCHES.	0.00635	0.01435	0.02350	0.03250

TABLE - III

THEORETICAL AND EXPERIMENTAL STRESSES FOR NORMAL REACTIONS LOCATED AT 30° ANGLES WITH THE VERTICAL.

ANGLE IN DEGREES		0	45	90	135	180	225	270	315
LOAD IN LBS.	STRESS IN PSI.								
100	THEORITICAL	-146	-13.64	+171.6	-23.58	-809	-23.58	+171.6	-13.64
	EXPERIMENTAL	-158.2	-38.58	+132.4	-71.7	-23.72	-54.4	+134	-32.14
200	THEORETICAL	-292	-27.78	+343.2	-47.16	-1618	-47.16	+343.2	-27.28
	EXPERIMENTAL	-319.8	-71.7	+312	-112.8	-98.8	-100.3	+323.9	-50.9
300	THEORETICAL	-438	-40.92	+514.8	-70.74	-2427	-70.74	+514.8	-40.92
	EXPERIMENTAL	-477	-103.8	+489	-168	-138.4	-150.2	+522	-70.7
400	THEORETICAL	-584	-54.56	+686.4	-94.32	-3236	-94.32	+686.4	-54.56
	EXPERIMENTAL	-645	-138.5	+690	-204.5	-211	-197.6	+734	-107.2
500	THEORETICAL	-730	-68.20	+858	-117.9	-4045	-117.9	+858	-68.20
	EXPERIMENTAL	-809	-163	+890	-254.6	-299.6	-270	+914	-142.2
600	THEORETICAL	-876	-81.84	+1029.6	-141.48	-4854	-141.48	+1029.6	-81.84
	EXPERIMENTAL	-960	-191.8	+1105	-295.8	-374.4	-323	+1132	-172

+ Tensile Stress and - Compressive Stress.

TABLE - IV

THEORETICAL AND EXPERIMENTAL DEFLECTIONS FOR NORMAL REACTIONS LOCATED AT 30° ANGLES WITH THE VERTICAL.

LOAD IN LBS.	119	219	319	419	519	619
THEORETICAL DEFLECTION IN INCHES.	0.00999	0.01860	0.02700.	0.03550	0.04290	0.05240
EXPERIMENTAL DEFLECTION IN INCHES.	0.01100	0.02300	0.03450	0.04750	0.06100	0.07450

APPENDIX-A

TRANSVERSE SHEAR STRESS AND CIRCUMFERENTIAL COMPRESSION ON SHELL.

1. Transverse Shear Stress.

This developement follows that shown in Reference 1.

A short length cut from a cylindrical thin shell loaded as a beam is shown in Figure 6. Moments on the transverse sections are represented by M at AB and M + dM at CD. The total transverse shear J on AB is one half of the total load P.

To determine the unit shear at any point on the transverse section of the shell, consider the portion NM cut from the ring ABCD of the shell by radial planes making angles θ and $-\theta$ with the vertical. Treating this portion NM as a free body, the sum of all forces is zero.

If S is the total longitudinal shear on the section N and M, and f is the flexure stress then,

$$\sum fdA + \sum dfdA - \sum fdA + 2S = 0$$

$$\sum dfdA + 2S = 0$$

Now we know that $f = My/I$.

$$\frac{df}{dx} = \frac{dMy}{dxI} = \frac{Jy}{I}$$

Now, for a thin ring;

$$I = \pi r^3 t$$

Also,

$$y = r\cos\lambda$$

$$dA = tds = trd\lambda$$

But,

$$dF = dfdA$$

$$= Jydxtrd\lambda / I$$

$$= JrCos\lambda\, dxtrd\lambda / \pi r^3 t$$

$$\frac{dF}{dx} = \frac{JCos\lambda\, d\lambda}{\pi r}$$

$$\Delta F = \int_{-\theta}^{\theta} \frac{JCos\lambda\, d\lambda}{\pi r} = \frac{2JSin\theta}{\pi r}$$

ΔF is the change in the longitudinal force on the portion NM, per unit length of the ring. The longitudinal shear on unit length is balanced by this force.

$$2tS' = 2JSin\theta / \pi r.$$

Where S' is unit shear stress,

$$S' = JSin\theta / \pi rt.$$

But the shear stress has the same intensity on adjacent edges of a rectangular element ,therefore the unit shear stress on the ends of the free body NM at the points N and M also equals $JSin\theta / \pi rt$, and is directed normal to the radial planes; it is tangent to the shell.

If S is the transverse tangential shear force per unit length arc, then

$$S = JSin\theta / \pi r$$

This shear is tangent to the shell at all points, and its values are zero at the top and bottom of the shell. The maximum value is at mid-depth, and its value is

$J/\pi r$, twice the average value over the entire section.

2. Circumferential Compression.

The equation for the normal force N_α in terms of total load P and an angle θ as shown in Figure 2 can be found as follows:

$$\text{Since, } J = P/2$$

$$S = P\sin\theta/2\pi r$$

The sum of the transverse shear forces acting on both transverse sections AB and CD of the shell will be:

$$S = P\sin\theta/\pi r$$

This shear produces a radial pressure at the interface of the shell and the ring equal to N_α lbs.per unit length of arc.These combined forces produce a circumferential compression of T lbs.,the total compression on any radial section making an angle θ with the vertical.

In Figure 3, any element $rd\theta$ of the shell is in equilibrium under the action of the forces shown. The equilibrium moment about the center of the shell is zero. Taking moments about the center of the shell,we get:

$$Tr + \frac{P\sin\theta}{\pi r}dsr - (T + dT)r = 0 \qquad (1)$$

$$dT = P\sin\theta ds/\pi r$$

$$T = \int_0^\theta P\sin\theta r d\theta/\pi r$$

$$T = \frac{P}{\pi}\left[1 - \cos\theta\right] \qquad \text{(2)}$$

Now $T = N_{\alpha}r$

$$N_{\alpha} = \frac{P}{\pi r}\left[1 - \cos\theta\right] \qquad \text{(3)}$$

This pressure is per unit length of arc.

This normal pressure distribution is zero when $\theta=0$ and maximum when $\theta = 180^{\circ}$. This normal pressure distribution is symmetrical about the vertical axis of the shell. The normal and tangential components of the support reactions are also equal and symmetrical about the vertical axis of the shell.

APPENDIX-B

MOMENT EQUATION FOR SUPPORT REACTIONS HAVING NORMAL AND TANGENTIAL COMPONENTS.

For this problem, the forces are shown in Figure 2 and the "reduced system" is shown in Figure 5.

In Figure 5, taking moments about point 1 at $\phi=0$,

$$\left\{ M_{\alpha} - \left[-2\frac{P\cos\delta}{2\cos(\delta-\gamma)}\left(\frac{r}{2\pi}\right) + \int_0^{2\pi} \left(\frac{r}{2\pi}\right) N_{\alpha}\, ds \right] \right.$$

$$- \int_0^{2\pi} \left(\frac{\phi}{2\pi}\right) N_{\alpha}\, ds\,(r\sin\phi)$$

$$+ \left[\frac{P\cos\delta}{2\cos(\delta-\gamma)}\left(\frac{\pi-\gamma-\alpha}{2\pi}\right) - \left(\frac{\mu}{2\pi}\right)\frac{P\sin\delta}{2\cos(\delta-\gamma)} \right]\Big(r\sin(\alpha+\gamma)\Big)$$

$$- \left[\frac{P\cos\delta}{2\cos(\delta-\gamma)}\left(\frac{\pi+\gamma-\alpha}{2\pi}\right) + \left(\frac{\mu}{2\pi}\right)\frac{P\sin\delta}{2\cos(\delta-\gamma)} \right]\Big(r\sin(\gamma-\alpha)\Big)$$

$$- \left[\frac{P\sin\delta}{2\cos(\delta-\gamma)}\left(\frac{\pi-\gamma-\alpha}{2\pi}\right)\Big(r + r\cos(\alpha+\gamma)\Big) \right]$$

$$\left. + \left[\frac{P\sin\delta}{2\cos(\delta-\gamma)}\left(\frac{\pi+\gamma-\alpha}{2\pi}\right)\Big(r + r\cos(\gamma-\alpha)\Big) \right] \right\} = 0$$

Now $ds = r\,d\phi$

and $N_{\alpha} = \dfrac{P}{r\pi}\left[1 - \cos\theta\right]$

and $\theta = \alpha + \phi$.

On substituting in the above equation and simplifing we get:

$$M = -\frac{Pr\cos\delta}{2\pi\cos(\delta-\gamma)} + \frac{\frac{Pr(2\ )}{2}}{2\pi} + \frac{\frac{Pr}{2}}{2\pi}\left(-2\pi + \frac{\pi}{2}\cos\alpha + \pi\sin\alpha\right)$$

$$-\frac{Pr}{4\pi\cos(\delta-\gamma)}\left\{\cos\delta\Big[(\pi-\gamma-\alpha)\sin(\alpha+\gamma) - (\pi+\gamma-\alpha)\sin(\gamma-\alpha)\Big] - \mu\sin\delta\Big[\sin(\alpha+\gamma) + \sin(\gamma-\alpha)\Big] + \sin\delta\Big[(\pi+\gamma-\alpha) - (\pi-\gamma-\alpha) + (\pi+\gamma-\alpha)\cos(\gamma-\alpha) - (\pi-\gamma-\alpha)\cos(\alpha+\gamma)\Big]\right\}$$

$$M = -\frac{Pr}{2\pi}\Bigg[-\frac{1}{2}\cos\alpha - \pi\sin\alpha + \frac{\cos\delta}{\cos(\delta-\gamma)}\Big(1 + \pi\cos\gamma\sin\alpha - \gamma\sin\gamma\cos\alpha - \alpha\cos\gamma\sin\alpha\Big) + \frac{\sin\delta}{\cos(\delta-\gamma)}\Big(-\mu\sin\gamma\cos\alpha + \gamma + \pi\sin\gamma\sin\alpha + \gamma\cos\gamma\cos\alpha - \alpha\sin\gamma\sin\alpha\Big)\Bigg]$$

APPENDIX - C

DEFLECTION EQUATION FOR SUPPORT REACTIONS HAVING NORMAL AND TANGENTIAL COMPONENTS.

From equation 4, Part C-b, Section II and the moment M_α of Appendix B, we can write:

$$\frac{d^2U}{d\alpha^2} + U = -K\Bigg[\frac{\cos\delta}{\cos(\delta-\gamma)} - \frac{1}{2}\cos\alpha - \pi\sin\alpha$$

$$+\frac{\cos\delta}{\cos(\delta-\gamma)}\Big(\pi\cos\gamma\sin\alpha - \gamma\sin\gamma\cos\alpha$$

$$-\alpha\cos\gamma\sin\alpha\Big) + \frac{\sin\delta}{\cos(\delta-\gamma)}\Big(-\mu\sin\gamma\cos\alpha$$

$$+\gamma+\pi\sin\gamma\sin\alpha + \gamma\cos\gamma\cos\alpha - \alpha\sin\gamma\sin\alpha\Big)\Bigg]$$

Where $K = Pr^3/2\pi EI$

The solution of the above equation can be written as follow:

$$U = U_c + U_p$$

Where $U_c = A\cos\alpha + B\sin\alpha$

U_p can be found from the following,

$$(D^2+1)U_p = -K\Bigg[\frac{\cos\delta}{\cos(\delta-\gamma)} - \frac{1}{2}\cos\alpha - \pi\sin\alpha$$

$$+\frac{\cos\delta}{\cos(\delta-\gamma)}\Big(\pi\cos\gamma\sin\alpha - \gamma\sin\gamma\cos\alpha$$

$$-\alpha\cos\gamma\sin\alpha\Big) + \frac{\sin\delta}{\cos(\delta-\gamma)}\Big(-\mu\sin\gamma\cos\alpha$$

$$+\gamma+\pi \operatorname{Sin}\gamma \operatorname{Sin}\alpha + \gamma \operatorname{Cos}\gamma \operatorname{Cos}\alpha - \alpha \operatorname{Sin}\gamma \operatorname{Sin}\alpha \Big)\Big]$$

Where $\frac{d}{d\alpha} = D$

Solving U_p from the above equation we get:

$$U_p = -K\left[\frac{\operatorname{Cos}\delta}{\operatorname{Cos}(\delta-\gamma)} - \frac{1}{2}\left(\frac{\alpha}{2}\operatorname{Sin}\alpha\right) - \pi\left(-\frac{\alpha}{2}\operatorname{Cos}\alpha\right)\right.$$

$$+\frac{\operatorname{Cos}\delta}{\operatorname{Cos}(\delta-\gamma)}\Big(\pi\operatorname{Cos}\gamma\left(-\frac{\alpha}{2}\operatorname{Cos}\alpha\right) - \gamma\operatorname{Sin}\gamma\left(\frac{\alpha}{2}\operatorname{Sin}\alpha\right)$$

$$-\operatorname{Cos}\gamma\left(-\frac{1}{4}\alpha^2\operatorname{Cos}\alpha + \frac{1}{4}\alpha\operatorname{Sin}\alpha\right)\Big)$$

$$+\frac{\operatorname{Sin}\delta}{\operatorname{Cos}(\delta-\gamma)}\Big(-\mu\operatorname{Sin}\gamma\left(\frac{\alpha}{2}\operatorname{Sin}\alpha\right) + \gamma$$

$$+\pi\operatorname{Sin}\gamma\left(\frac{-1}{2}\alpha\operatorname{Cos}\alpha\right) + \gamma\operatorname{Cos}\gamma\left(\frac{\alpha}{2}\operatorname{Sin}\alpha\right)$$

$$\left.-\operatorname{Sin}\gamma\left(-\frac{1}{4}\alpha^2\operatorname{Cos}\alpha + \frac{1}{4}\alpha\operatorname{Sin}\alpha\right)\Big)\right]$$

On substituting the values of U_c and U_p in the equation ,

$$U = U_c + U_p$$

We get;

$$U = A\cos\alpha + B\sin\alpha - K\left[\frac{\cos\delta}{\cos(\delta-\gamma)} - \frac{1}{4}\alpha\sin\alpha + \frac{\pi\alpha}{2}\cos\alpha\right.$$

$$+\frac{\cos\delta}{\cos(\delta-\gamma)}\left(-\frac{\pi\alpha}{2}\cos\gamma\cos\alpha - \frac{\gamma\alpha}{2}\sin\gamma\sin\alpha\right.$$

$$\left.+\frac{\alpha^2}{4}\cos\gamma\cos\alpha - \frac{\alpha}{4}\cos\gamma\sin\alpha\right)$$

$$+\frac{\sin\delta}{\cos(\delta-\gamma)}\left(-\frac{\alpha\mu}{2}\sin\gamma\sin\alpha + \gamma - \frac{\pi\alpha}{2}\sin\gamma\cos\alpha\right.$$

$$\left.\left.+\frac{\gamma\alpha}{2}\cos\gamma\sin\alpha + \frac{\alpha^2}{4}\sin\gamma\cos\alpha - \frac{\alpha}{4}\sin\gamma\sin\alpha\right)\right]$$

Solving above equation for constants A and B with following boundary conditions we get:

$$B = K\left[\frac{\pi}{2} - \frac{\pi\cos\delta\cos\gamma}{2\cos(\delta-\gamma)} - \frac{\pi\sin\delta\sin\gamma}{2\cos(\delta-\gamma)}\right] \quad \text{for } \frac{dU}{d\alpha}=0 \text{ at } \alpha=0.$$

$A = KR$, for $U = 0$ at $\alpha = \pi - \gamma$

Where
$$R = \frac{1}{\cos\gamma}\left\{\left[\frac{\pi}{2} - \frac{\pi\cos\delta\cos\gamma}{2\cos(\delta-\gamma)} - \frac{\pi\sin\delta\sin\gamma}{2\cos(\delta-\gamma)}\right]\sin\gamma\right.$$

$$-\left[\frac{\cos\delta}{\cos(\delta-\gamma)} - \left(\frac{\pi-\gamma}{4}\right)\sin\alpha - \pi\left(\frac{\pi-\gamma}{2}\right)\cos\gamma\right.$$

$$+\frac{\cos\delta}{\cos(\delta-\gamma)}\left(\pi\left(\frac{\pi-\gamma}{2}\right)\cos\gamma\cos\gamma - \gamma\left(\frac{\pi-\gamma}{2}\right)\sin\gamma\sin\gamma\right.$$

$$\left.-\left(\frac{\pi-\gamma}{4}\right)^2\cos\gamma\cos\gamma - \left(\frac{\pi-\gamma}{4}\right)\cos\gamma\sin\gamma\right)$$

$$+\frac{\sin\delta}{\cos(\delta-\gamma)}\left(-\mu\left(\frac{\pi-\gamma}{2}\right)\sin\gamma\sin\gamma + \gamma\right.$$

$$+ \pi\left(\frac{\pi - \gamma}{2}\right)\sin\gamma\cos\gamma + \gamma\left(\frac{\pi - \gamma}{2}\right)\cos\gamma\sin\gamma$$

$$- \left(\frac{\pi - \gamma}{4}\right)^2\sin\gamma\cos\gamma - \left(\frac{\pi - \gamma}{4}\right)\sin\gamma\sin\gamma\Big)\Big]\Big\}$$

On substituting the above values of A and B in equation for radial deformation we get;

$$U = K\left\{R\cos\alpha + \left(\frac{\pi}{2} - \frac{\pi\cos\delta\cos\gamma}{2\cos(\delta-\gamma)} - \frac{\pi\sin\delta\sin\gamma}{2\cos(\delta-\gamma)}\right)\sin\alpha\right.$$

$$- \left[\frac{\cos\delta}{\cos(\delta-\gamma)} - \frac{\alpha}{4}\sin\alpha + \frac{\pi\alpha}{2}\cos\alpha\right.$$

$$+ \frac{\cos\delta}{\cos(\delta-\gamma)}\left(-\frac{\pi\alpha}{2}\cos\gamma\cos\alpha - \frac{\gamma\alpha}{2}\sin\gamma\sin\alpha\right.$$

$$\left. + \frac{\alpha^2}{4}\cos\gamma\cos\alpha - \frac{\alpha}{4}\cos\gamma\sin\alpha\right) + \frac{\sin\delta}{\cos(\delta-\gamma)}\Big(\gamma$$

$$- \frac{\mu\alpha}{2}\sin\gamma\sin\alpha + \frac{\gamma\alpha}{2}\cos\gamma\cos\alpha + \frac{\alpha^2}{4}\sin\gamma\cos\alpha$$

$$\left.\left. - \frac{\alpha}{4}\sin\gamma\sin\alpha - \frac{\pi\alpha}{2}\sin\gamma\cos\alpha\Big)\right]\right\}$$

APPENDIX - D

FORMULA FOR STRESS IN TERMS OF MOMENT.

From Section II, Part D, Equation 11, we have:

$$\sigma_o = \frac{-M_{\alpha}\left(\frac{h}{2} + e\right)}{A.e.r_o}$$

For the Experimental Model we have:

$h=0.762''$

$r_o=5.637''$

$b=2''$

$A = h.b = 1.524 \text{ inch}^2$

$r = 5.256''$

$$e \approx \frac{h^2}{12r}\left[1 + \frac{h^2}{15r^2}\right]$$

$e \approx 0.00921''$

On substituting these values in the above equation for stress, we get:

$$\sigma_o = -4.94\, M_{\alpha}$$

REFERENCES

1. TESTS OF CYLINDRICAL SHELLS
by W.M.Wilson and E.D.Alson,
Illinois Engineering Experiment Station Bulletin, Series No.331,Published by The University of Illinois.

2. ENGINEERING DYNAMICS,VOLUME II:
ELASTIC PROBLEMS OF SINGLE MACHINE ELEMENTS,
by C.B.Biezeno and R.Grammel,
Blackie and Son Limited,London and Glasgow,1956,pp.131-143.

3. HAND-BOOK OF EXPERIMENTAL STRESS ANALYSIS
by M.Hetényi,
John Wiley and Sons,New York,1950.

4. FORMULAS FOR STRESS AND STRAIN,
by R.J.Roark,
McGraw-Hill Book Company Inc;New York,1954.

5. STRENGTH OF MATERIALS,PART I:
ELEMENTARY THEORY AND PROBLEMS,
by S.Timoshenko,
D.Van Nostrand Company,Inc;New York,1955.

6. STRESS IN RING STIFFNERS IN CYLINDERS
by W.S.Rumman,J.of the Structural Division,
A.S.C.E.,V.87,No.8,Dec.1961.

7. PRINCIPLES OF EXPERIMENTAL STRESS ANALYSIS
by M.B.Moore,
Prentice-Hall,Inc;New York, 1954.

8. ADVANCED MECHANICS OF MATERIALS, EDITION 2.
by Seely and Smith,
John Wiley & Sons Inc; New York,1952.

9. INTRODUCTION TO THEORETICAL AND EXPERIMENTAL ANALYSIS OF STRESS AND STRAIN
by A.J.Durell, E.A.Phillips, and C.H.Tsao,
McGraw-Hill Book Company, Inc; New York, 1958.

10. ELECTRIC RESISTANCE STRAIN GAGES
by W.B.Dobie and P.C.G.Isaac,
The English University Press Limited,London,1958.

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